Inward Transport of Toroidal Momentum in Tokamak Plasmas

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Abstract

A theory of inward toroidal momentum transport in tokamak plasmas is presented. An off-diagonal component in the toroidal momentum flux due to acoustic wave coupling gives rise to the inward momentum transport. This mechanism requires a finite spectral average parallel wavenumber, $\left\langle k_{\|} \right\rangle \neq 0$. Standard drift wave modes are symmetric about the rational surface and thus have $\left\langle k_{\|} \right\rangle = 0$. Radial electric field shear, however, induces a mode shift away from the rational surface and thus produces a finite spectral average parallel wavenumber. The toroidal flow generated can enhance the radial electric field shear through the radial force balance equation. This describes a feedback mechanism in which the toroidal flow and radial electric field shear enhance each other.

Outline

I. The toroidal momentum equation

II. Observations of inward toroidal momentum transport

III. Previous theoretical work

IV. Inward transport of toroidal momentum

V. Feedback mechanism

VI. Discussion and Conclusions

I. Toroidal Momentum Equation

$$\frac{d}{dt}\left\langle V_\phi \right\rangle + \frac{d}{dr}\left\langle \tilde{V}_r \tilde{V}_\phi \right\rangle = S_{V_\phi}$$

where

$\left\langle V_\phi \right\rangle$: mean toroidal flow

S_{V_ϕ}: source/sink of toroidal flow (e.g., neutral beam injection, collisional flow damping)

$nm_i \left\langle \tilde{V}_r \tilde{V}_\phi \right\rangle$: toroidal momentum flux

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A common assumption is that the flux is diffusively outward,

$$\left\langle \tilde{V}_r \tilde{V}_\phi \right\rangle = -\chi_\phi \frac{d}{dr}\left\langle V_\phi \right\rangle$$

and that in the absence of toroidal momentum sources, there would be no toroidal flow.

II. Observations

There is observational evidence, however, that tokamak plasmas can spontaneously generate toroidal flow.

- Neutral beam heated plasmas on JFT-2M
  (*Ida, et al., Phys. Rev. Lett., 1995*)

  Observed spontaneous plasma rotation in a direction antiparallel to the plasma current in experiments in which the beam directions were reversed.

- RF heated plasmas on Alcator C-Mod
  (*Rice, et al., Nucl. Fusion., 1998*)

  Observed spontaneous co-current plasma rotation in H-mode discharges "that have no direct momentum input."

- Ohmic plasmas on Alcator C-Mod
  (*Rice, et al., Phys. Rev. Lett., 2000*)

  Core plasma rotation reverses from counter to co-current in the transition from L to H mode in Ohmically heated discharges.

Some experiments (TFTR, namely) have observations indicating no inward transport of toroidal momentum.

III. Previous Theoretical Work

Theoretical work on the toroidal momentum flux has indicated the possibility of inward terms due to off-diagonal components in the flux (i.e., terms in the momentum flux proportional to gradients other than the toroidal momentum gradient).

- $\langle k_y k_\parallel \rangle \neq 0$ leads to an inward componenent in the flux (where k_y and $k_\parallel$ are the poloidal and parallel wavenumbers).
  (*Ware, et al., Phys. Fluids B, 1992*)

- $\langle k_y k_\parallel \rangle \neq 0$ is a direct consequence of the effect of radial electric field shear on the mode structure.
  (*Diamond, et al., IAEA, 1994*)

More recently work has been done on possible RF induced sources of toroidal momentum (*Perkins, 2000*) and a neoclassical quasilinear calculation (*Shaing, 2000*) which suggests $\langle k_\parallel \rangle \neq 0$ due to q-profile effects, but these mechanisms are unlikely to produce the observed rates of toroidal flow.

IV. Toroidal Momentum Flux

To calculate the toroidal flux, we use the relation between the toroidal and parallel flow,

$$V_\parallel = \frac{B_\phi}{B} V_\phi + \frac{B_\theta}{B} V_\theta \quad \rightarrow \quad V_\phi \approx V_\parallel - \frac{B_\theta}{B} V_\theta$$

The fluctuating parallel flow equation:

$$\frac{d}{dt}\tilde{V}_\parallel = -\tilde{V}_r \frac{d}{dr}\left\langle V_\parallel \right\rangle - \nabla_\parallel \tilde{p} + \tilde{E}_\parallel + \nu \nabla^2 \tilde{V}_\parallel$$

$$R_{\mathbf{k}}^{-1} \tilde{V}_{\parallel_{\mathbf{k}}} = -\tilde{V}_{r_{\mathbf{k}}} \frac{d}{dr}\left\langle V_\parallel \right\rangle - ik_\parallel \left(\tilde{\phi}_{\mathbf{k}} + \tilde{p}_{\mathbf{k}} \right)$$

where $R_{\mathbf{k}}^{-1} = -i\omega + 1/\tau_c$ is the response function.

We can use these to calculate the toroidal Reynolds stress:

$$\left\langle \tilde{V}_r \tilde{V}_\parallel \right\rangle = -\chi_\phi \frac{d}{dr}\left\langle V_\parallel \right\rangle + \sum_{\mathbf{k}} k_\theta k_\parallel R_{\mathbf{k}} \left\langle \tilde{\phi}_{-\mathbf{k}} \left(\tilde{\phi}_{\mathbf{k}} + \tilde{p}_{\mathbf{k}} \right) \right\rangle$$

$$\left\langle \tilde{V}_r \tilde{V}_\phi \right\rangle = \left\langle \tilde{V}_r \tilde{V}_\parallel \right\rangle - \frac{B_\theta}{B} \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle$$

where $\chi_\phi = \sum_{\mathbf{k}} R_{\mathbf{k}} \left\langle V_{r_{\mathbf{k}}}^2 \right\rangle$ is the anomalous viscosity.

Toroidal Reynolds stress:

$$\left\langle \tilde{V}_r \tilde{V}_\phi \right\rangle = -\chi_\phi \frac{d}{dr}\left\langle V_\phi \right\rangle + \sum_{\mathbf{k}} k_\theta k_\parallel R_{\mathbf{k}} \left\langle \tilde{\phi}_{-\mathbf{k}} \left(\tilde{\phi}_{\mathbf{k}} + \tilde{p}_{\mathbf{k}} \right) \right\rangle$$
$$- \frac{B_\theta}{B} \left[\chi_\phi \frac{d}{dr}\left\langle V_\phi \right\rangle + \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \right]$$

where the terms are

$$\text{Outward radial diffusion:} \quad -\chi_\phi \frac{d}{dr}\left\langle V_\phi \right\rangle$$
$$\text{Off-diagonal pinch term:} \quad \sum_{\mathbf{k}} k_\theta k_\parallel R_{\mathbf{k}} \left\langle \tilde{\phi}_{-\mathbf{k}} \left(\tilde{\phi}_{\mathbf{k}} + \tilde{p}_{\mathbf{k}} \right) \right\rangle$$
$$\text{Poloidal flow terms:} \quad -\frac{B_\theta}{B} \left[\chi_\phi \frac{d}{dr}\left\langle V_\phi \right\rangle + \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \right] \approx 0$$

The off-diagonal pinch term:

$$\Gamma_\phi = \sum_{\mathbf{k}} k_\theta k_\parallel R_{\mathbf{k}} \left\langle \tilde{\phi}_{-\mathbf{k}} \left(\tilde{\phi}_{\mathbf{k}} + \tilde{p}_{\mathbf{k}} \right) \right\rangle$$
$$= \sum_{\mathbf{k}} k_\theta k_\parallel R_{\mathbf{k}} \chi_{\mathbf{k}}^{ion} \left\langle \tilde{\phi}_{\mathbf{k}}^2 \right\rangle$$

- The pinch term requires $\left\langle k_y k_\parallel \right\rangle \neq 0$.
- What can produce this effect?

Radial electric field shear shifts drift wave eigenmodes off of the rational surface (*Carreras, et al., Phys. Fluids B, 1992*):

$$\tilde{\phi}_{\mathbf{k}}(x) = \tilde{\phi}_{\mathbf{k}0} \exp\left[-\left(x - \alpha_{\mathbf{k}}\right)^2 / 2W_0^2\right]$$

$$\uparrow$$

mode shift

where

$$\alpha_{\mathbf{k}} = \frac{\omega_* L_s^2}{2Bv_{Ti}^2 k_\theta} E_r'$$

Thus, the finite spectral average parallel wave number is dependent on the shear in the radial electric field,

$$\left\langle k_y k_\| \right\rangle = \frac{v_* L_s}{2Bv_{Ti}^2} \left\langle k_\theta^2 \right\rangle E_r'$$

so that the pinch term is also linearly dependent on the radial electric field shear,

$$\Gamma_\phi = \left(\frac{v_* L_s}{2Bv_{Ti}^2} E_r'\right) \sum_{\mathbf{k}} \left\langle k_\theta^2 \right\rangle R_{\mathbf{k}} \chi_{\mathbf{k}}^{ion} \left\langle \tilde{\phi}_{\mathbf{k}}^2 \right\rangle$$

V. Feedback Mechanism

So radial electric field shear produces a mechanism for the generation of toroidal flow... the toroidal flow will also contribute to the generation of radial electric field.

The radial force balance equation:

$$E_r = \frac{1}{en}\frac{d}{dr}p - v_\theta B_\phi + v_\phi B_\theta$$

$$E_r' = \frac{1}{en}p'' - \frac{1}{en^2}n'p' - (v_\theta B_\phi)' - (v_\phi B_\theta)'$$

- The pressure gradient or poloidal flow can provide a seed E_r' for generation of toroidal flow.
- The feedback mechanism is based on the toroidal flow dependence of E_r':

$$E_r' \approx (v_\phi B_\theta)'$$

An important point is that the feedback to the radial electric field gradient is dependent not only on v_ϕ' but also B_θ'. See (*Rice, et al., Nucl. Fusion., 1998*) for an example where v_ϕ peaks on axis while E_r is peaked off axis.

VI. Discussion and Conclusions

A mechanism for an anomalous pinch of toroidal momentum has been shown.

- A finite spectral average parallel wavenumber produces an off-diagonal term in the toroidal momentum flux.
- Radial electric field shear can produce a radial mode shift which results in a finite parallel wavenumber.
- The toroidal flow generated will feedback on the radial electric field through the force balance equation.

The toroidal flow profile that develops will be largely independent of the fluctuation levels as long as the anomalous viscosity is the dominant flow damping mechanism,

$$-\chi_\phi d \left\langle V_\phi \right\rangle / dr + \Gamma_\phi \approx 0$$