Suppression of Transport by Alignment of Fluctuation Phases in Strongly Sheared Flow

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The Bottom Line:

In strong E×B flow shear, transport of a scalar is reduced most strongly by phase alignment of the scalar fluctuations and the electrostatic potential fluctuations.

Outline

- Definition of the cross phase
- Experimental measurements
- Previous theory work on cross phase
- Transport reduction due to cross phase modification in strong E×B shear
- Conclusions and ongoing work

Definition of the Cross Phase

- Consider the electrostatic particle flux:

$$\Gamma = \langle \tilde{v}\tilde{n} \rangle = \sum_{\mathbf{k}} k_y |\tilde{\phi}_{-\mathbf{k}}| |\tilde{n}_{\mathbf{k}}| \sin\delta_{\mathbf{k}}$$

  - $\tilde{n}_{\mathbf{k}}$ is the density fluctuation
  - $\tilde{\phi}_{\mathbf{k}}$ is the electrostatic potential fluctuation
  - $\delta_{\mathbf{k}}$ is the cross phase between density and electrostatic potential fluctuations

A coherency of ~1 has been assumed.

Phase alignment of n and ϕ results in transport suppression

- Phase alignment: $\delta_{\mathbf{k}} \rightarrow 0$

  $\Gamma \quad \sin\delta_{\mathbf{k}} \rightarrow 0$

- Note for comparison with experiment:

n-ϕ phase	n-E_r phase	n-v phase
$\delta_{\mathbf{k}}$	$\alpha_{\mathbf{k}} = \delta_{\mathbf{k}} - \frac{\pi}{2}$	$\alpha_{\mathbf{k}}$

ExB flow shear key effect in all transport barriers with $\omega_{ExB} > \gamma$

- Transport reduction that establishes existence of a barrier (not a fluctuation level drop or change in correlation length)
- How does ExB flow shear, v_0', suppress transport?
  - Suppressed amplitudes or phase alignment

Experimental evidence of cross phase modification is abundant

- Reduced transport due to phase modification measured on a number of different toroidal confinement devices:
  - CCT Tokamak Tynan, et al., PRL (1992)
  - DIII-D Tokamak Moyer, et al., Phys. Plas. (1995)
  - TEXTOR Tokamak Boedo, et al., PRL (2000)
  - H-1 Heliac Shats, et al., PRL (2000)
  - RFX Rev. Field Pinch Antoni, et al., PPCF (2000)
  - MST Rev. Field Pinch Sarff, et al., Cz. J. Phys. (2000)

Experiment: flux strongly suppressed relative to fluctuation amplitudes

- Measurements suggest that ExB shear affects both fluctuation amplitudes and cross phases
- In some cases, the effect on the cross phase is much stronger than the effect on the amplitudes

Previous theoretical work on the cross phase has focused on weak E_r shear

- g-mode turbulence model
  - ➢ Simulations by Carreras, et al., '94
  - ➢ Analytic work by Ware, et al., '96, '98
    - ✷ Analytic model solved in the weak shear limit:

$$v_0' \Delta x k_y \quad << \quad \gamma \quad or \quad D / \Delta x^2$$

ExB shearing rate

linear growth rate

nonlinear decorrelation rate

In the weak shear limit, amplitude & cross phase suppression are comparable

- Results:

$$\sin\delta_{\mathbf{k}} \approx 1 - 1.2\frac{\left(v_0'\Delta x k_y\right)^2}{\gamma^2}$$

$$|n_{\mathbf{k}}||\phi_{\mathbf{k}}| \approx 1 - 1.5\frac{\left(v_0'\Delta x k_y\right)^2}{\gamma^2}$$

  - Response of amplitude & cross phase to ExB flow shear is similar

Strong shear theory for the cross phase

- Generic, BDT-like treatment of the effect of ExB shear on the cross phase
- Valid in the strong shear limit,

$$v_0' \Delta x k_y \quad >> \quad \gamma \quad or \quad D / \Delta x^2$$

- Examines an arbitrary advected scalar, χ

Model Equation

- Advected scalar fluctuations:

$$\frac{\partial}{\partial t}\tilde{\chi} + v_0(x)\frac{\partial}{\partial y}\tilde{\chi} - \quad \tilde{\phi} \times \hat{z} \qquad \tilde{\chi} = \frac{\partial \tilde{\phi}}{\partial y}\frac{d\chi_0}{dx}$$

  - consider a linear shear flow:

$$v_0(x) = v_0(x) + (x - x_0)v_0'$$

  - examine the response of χ to ϕ, both amplitude and phase

Basic result can be understood heuristically

- Replace nonlinearity with anomalous diffusivity, $-\quad \tilde{\phi}\times\hat{z}\quad\quad \tilde{\chi} \rightarrow -d\frac{\partial^2}{\partial y^2}\tilde{\chi}$

$$\tilde{\chi}_{\mathbf{k}} = \frac{ik_y\left(d\chi_0/dx\right)}{-i\omega + ik_y v_0' x + k_y^2 d}\tilde{\phi}_{\mathbf{k}}$$

  - can examine this in two limits,

weak shear:

$$k_y^2 d >> k_y v_0' x$$

strong shear:

$$k_y v_0' x >> \omega, k_y^2 d$$

- Weak shear limit:

$$\tilde{\chi}_{\mathbf{k}} \approx \frac{ik_y(d\chi_0/dx)}{k_y^2 d}\tilde{\phi}_{\mathbf{k}} \qquad \rightarrow \qquad \delta_{\mathbf{k}} \approx \frac{\pi}{2}$$

  - ➢ Maximal transport to lowest order
  - ➢ Small $k_y v_0' x$ introduces small reduction

- Strong shear limit

$$\tilde{\chi}_{\mathbf{k}} = \frac{k_y(d\chi_0/dx)}{k_y v_0' x}\tilde{\phi}_{\mathbf{k}} \qquad \rightarrow \qquad \delta_{\mathbf{k}} \approx 0$$

  - ➢ No transport to lowest order
  - ➢ Small $k_y^2 d$ introduces small transport

In the strong shear limit, cross phase reduction is dominant mechanism

- Expansion in the strong shear limit:

$$\mathrm{Im}\left(\frac{\tilde{\chi}_{\mathbf{k}}}{\tilde{\phi}_{\mathbf{k}}}\right) \sim \underbrace{\frac{k_y (d\chi_0/dx)}{k_y v_0' x}}_{\text{amplitude factor}} \underbrace{\left(\frac{k_y^2 d}{k_y v_0' x}\right)}_{\text{cross phase factor}}$$

amplitude factor — cross phase factor

  - Nonlinear diffusivity, *d*, also reduced in strong shear … gives an additional factor of ~ $\left(k_y v_0' x\right)^{-2}$

Cross phase response to shear obtained by solving a renormalized equation for χ

- Advective operator is renormalized in the usual EDQNM fashion
- Renormalized operator solved using a Green function
  - Green function found with WKB analysis for the asymptotic regime of strong shear
  - Radial integral over the Green function is also evaluated using asymptotic methods

Radial scale of the Green function is the reduced correlation length of BDT

- Width of the Green function is the BDT shear reduced radial correlation length,

$$x/\Delta x \sim \left(d_{\mathbf{k}} / v_0' k_y \Delta x^3\right)^{1/3}$$

- A mixing layer, which emerges after the radial integral over the Green function, is a much narrower width (BDT cubed)
  - Systematic, ordered analysis that leads to the final result - not simply a heuristic estimate

Outside a narrow resonance layer, flow shear strongly suppresses flux

- Inside the resonance (mixing) layer, flow term vanishes:

  $v_0' k_y x \to 0$ large flux (like weak shear)

- Effect confined to a narrow layer: $$\frac{x}{\Delta x} \sim \frac{D_{\mathbf{k}}}{v_0' k_y \Delta x^3}$$

- Outside the resonance layer, flow shear term dominates the response

Scalar mixing layer is the narrowest structure in the turbulence

- Radial scales involved in the strong shear limit:

Structure	Width in strong shear	Role in Γ
Fluctuation Eigenmode, $\|\phi(x)\|^2$	$W \sim (v_0')^0$ (no shear dependence)	Flux where $\|\phi(x)\|^2$ is large
Response of χ to impulse, G	$\sim \left(d_{\mathbf{k}} / v_0' k_y \Delta x^3\right)^{1/3}$ (BDT width)	Integrates out of flux expression
Mixing layer χ	$\sim \left(d_{\mathbf{k}} / v_0' k_y \Delta x^3\right)^{1}$ (very narrow)	Flux strongly suppressed outside layer

Transport flux reduced by strong E×B flow shear

- Upper bound in strong flow shear limit:

$$\Gamma = -\sum_{\mathbf{k},\mathbf{k}'} \underbrace{\left(\frac{k_y^2 \Delta x^2 |\psi_{\mathbf{k}}|^2}{k_y \Delta x^3 v_0'} \right) \frac{d\chi_0}{dx}}_{\text{amplitude factor}} \underbrace{\left[\frac{|d\psi_{\mathbf{k}}/dx|^2 (\gamma_{\mathbf{k}} - \gamma_{\mathbf{k}'})}{(\Delta x v_0')^3 k_y (k_y - k_y')} \right]}_{\text{cross phase factor}}$$

  - Cross phase suppression much stronger than amplitude suppression
  - Have not calculated the eigenmode

Summary

- Strong ExB shear flow affects transport of a passive scalar most strongly through modification of the cross phase
- Physics mechanism:
  - Strong v_0' → $\tilde{\chi}$ in phase with $\tilde{\phi}$
  - Out of phase $\tilde{\chi}$ → requires anomalous dissipation, d
  - d also suppressed by strong shear

Ongoing Work

- Investigating the effect of magnetic shear on this calculation
  - Possible mechanism for a sign change in the cross phase - see P. W. Terry, Thursday 5:00pm
- Effect of E_r' on momentum transport
  - Expect momentum transport not suppressed