Self-Consistent Cross Phase Evolution

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Goals of this work

- Develop an evolution equation for the cross phase
- Use this equation in a self-consistent model for the L-H transition
- Estimate the effects on power thresholds and transition dynamics

The ***cross phase****, δ, between radial velocity and density (or temperature) fluctuations affects:*

- turbulent transport of particles (or heat)
- rates of nonlinear energy transfer due to the $E \times B$ nonlinearity
- timing of and threshold for L-H transitions in phase transition models

- Consider the cross phase between density and radial velocity fluctuations:

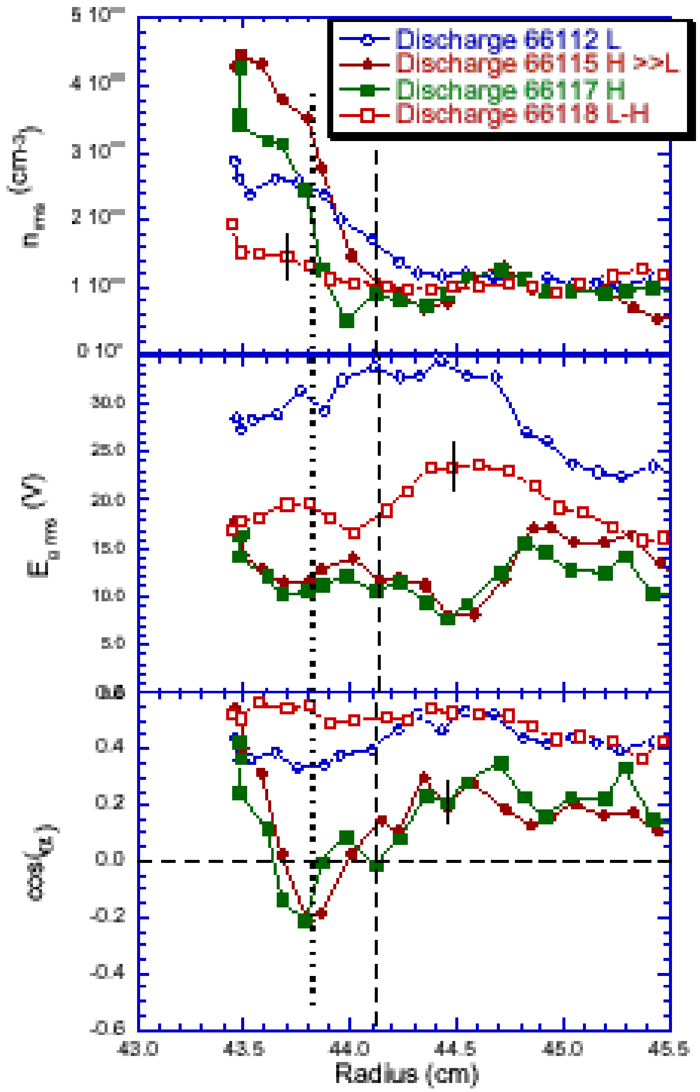
$$\Gamma = \langle \tilde{v}_r^* \tilde{n} \rangle = \langle \tilde{v}_r^2 \rangle^{1/2} \langle \tilde{n}^2 \rangle^{1/2} \; C_{v,n} \; \cos(\delta)$$

  - $C_{v,n}$ is the coherence (assume ~1)
  - δ is the cross phase

$$\cos\delta = \frac{\langle \tilde{v}_r^* \tilde{n} \rangle}{\langle \tilde{v}_r^2 \rangle^{1/2} \langle \tilde{n}^2 \rangle^{1/2}}$$

A number of experimental measurements have indicated that modification of the cross phase can be a large part of transport reduction in L-H transitions

- Tynan, et al. on CCT
- Moyer, et al. on DIII-D
- Boedo, et al. on DIII-D and TEXTOR
- Chapman, et al. on MST (coherence effect?)



- From Boedo, et al., PRL 2000

  Cross phase between temperature and electrostatic potential fluctuations

Results from numerical simulations of turbulence modeling the L-H transition has shown mixed results

- Carreras, et al., : Cross phase strongly modified by ExB shear in simulations of resistive pressure gradient driven turbulence
- Xue, et al., : Cross phase only weakly modified during L-H transitions in edge turbulence modeling

- The evolution of fluctuation amplitudes $\varepsilon \equiv \langle \tilde{n}^2 \rangle$ (i.e., an ``envelope'' equation) has been considered in many models:

$$\frac{d}{dt}\varepsilon = \gamma\varepsilon - \alpha\varepsilon^2 - \beta\varepsilon\left(\frac{dE}{dr}\right)^2$$

- We wish to consider an evolution equation for the cross phase:

$$\frac{d\delta_k}{dt} = \frac{i}{2}\frac{d}{dt}\ln\left(\frac{\langle n_k^* v_k \rangle}{\langle n_k v_k^* \rangle}\right)$$

III. Cross Phase Evolution

- Vorticity equation + density equation:

$$\frac{d}{dt} \quad {}^{2}\tilde{\varphi} + L_{\varphi\varphi}\tilde{\varphi} = L_{\varphi n}\tilde{n}$$

$$\frac{d}{dt}\tilde{n} + L_{nn}\tilde{n} = L_{n\varphi}\tilde{\varphi}$$

Total time derivatives Direct linear terms Cross linear terms

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \langle V_E \rangle \qquad + \quad \tilde{\varphi} \times \hat{z}$$

Mean convection Nonlinear convection

- The terms $L_{\varphi\varphi}, L_{\varphi n}, L_{nn}, L_{n\varphi}$, are operators that depend on the model
- Renormalized equations:

$$R_U \tilde{U} = L_{\varphi n}\tilde{n} - L_{\varphi\varphi}\tilde{\varphi} = L_{\varphi n}\tilde{n} - L'_{\varphi\varphi}\tilde{v}_r$$

$$R_n \tilde{n} = L_{n\varphi}\tilde{\varphi} \rightarrow L'_{n\varphi}\tilde{v}_r$$

- Use these to find cross phase equation:

$$\frac{d\delta_k}{dt} = \frac{i}{2}\left[\frac{1}{\langle n_k^* v_k\rangle}\frac{d}{dt}\langle n_k^* v_k\rangle - \frac{1}{\langle n_k v_k^*\rangle}\frac{d}{dt}\langle n_k v_k^*\rangle\right]$$

Cross phase evolution equation:

Nonlinear phase alignment — Quasilinear phase

$$\frac{d\delta}{dt} = -\left[\alpha_1 \langle \tilde{n}^2 \rangle - \alpha_2 (\quad n)\right] \sin\delta + \left[\alpha_3 \langle \tilde{n}^2 \rangle + \alpha_4 \langle V_E \rangle'^2\right] \cos\delta$$

Nonlinear phase shifts — ExB shear modification

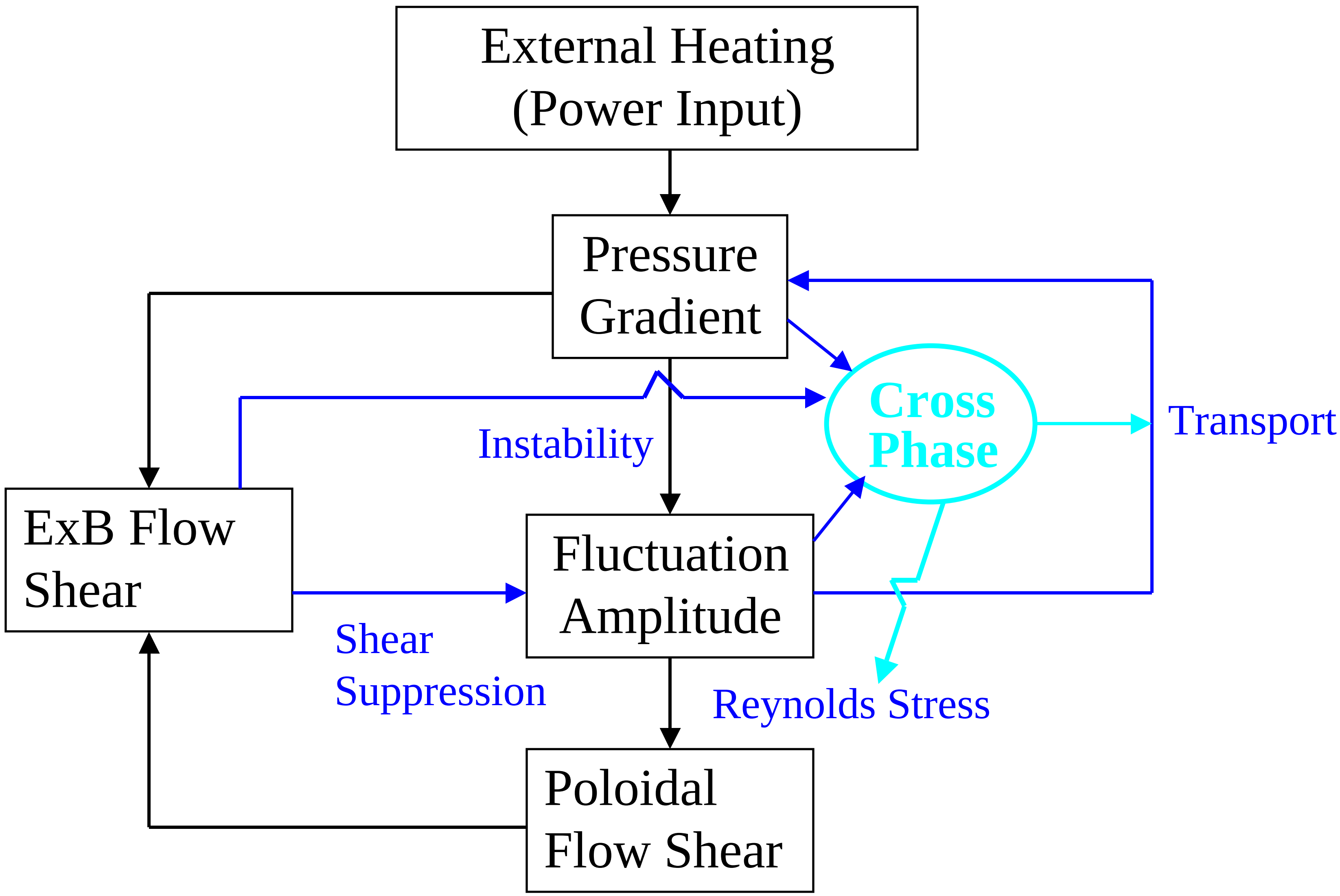
- The coefficients α_1, α_2, and α_3, are all model dependent

Properties of the cross phase equation:

- Fixed point for the cross phase:

$$\tan\delta = \frac{\left[\alpha_3\langle \tilde{n}^2\rangle + \alpha_4\langle V_E\rangle'^2\right]}{\left[\alpha_1\langle \tilde{n}^2\rangle - \alpha_2(\quad n)\right]}$$

- Nonlinear phase shifts and ExB shear flow tends to move $\tilde{n}$ and $\tilde{v}_r$ out of phase


External Heating
(Power Input)
Pressure
Gradient
Cross
Phase
Transport
Instability
ExB Flow
Shear
Fluctuation
Amplitude
Shear
Suppression
Reynolds Stress
Poloidal
Flow Shear

Predator-Prey Model with Cross Phase Evolution

(1) Fluctuation Amplitude Evolution:

Spectral transfer of energy

$$\frac{dE}{d\tau} = EN - k_1 E^2 \left(1 + k_8 \cos\delta\right) - EU^2$$

Linear instability

Shear suppression

(2) Poloidal Flow Shear Evolution:

Magnetic pumping (Poloidal flow damping)

$$\frac{dU}{d\tau} = k_2 EU \left(1 + k_9 \cos\delta\right) - k_3 U$$

Reynolds stress

(3) Pressure Gradient Evolution:

Collisional transport — Power input

$$\frac{dN}{d\tau} = -N - EN\cos\delta + Q$$

turbulent transport

(4) Radial Force Balance: $U = V - k_{10} N^2$

(5) Cross Phase Evolution:

Quasilinear phase — ExB shear modification

$$\frac{d\delta}{d\tau} = -k_{11} N \sin\delta - k_{12} E \sin\delta + k_{13} E \cos\delta + k_{14} V^2 \cos\delta$$

Nonlinear phase alignment — Nonlinear phase modification

E: Fluctuation Amplitude
N: Pressure Gradient
U: Poloidal Flow Shear
V: ExB Flow Shear
δ: Cross Phase

Q: Power Input
k_1-k_{10}: Physical Parameters

All equations are in dimensionless units

- The model has two nontrivial fixed points. Which is stable depends on the input power.
  - At low Q, a fixed point with U=0 is stable, this is the "L-mode" like fixed point.
  - At high Q, a fixed point with flow is stable, this is the "H-mode" like fixed point.

Input Parameters:

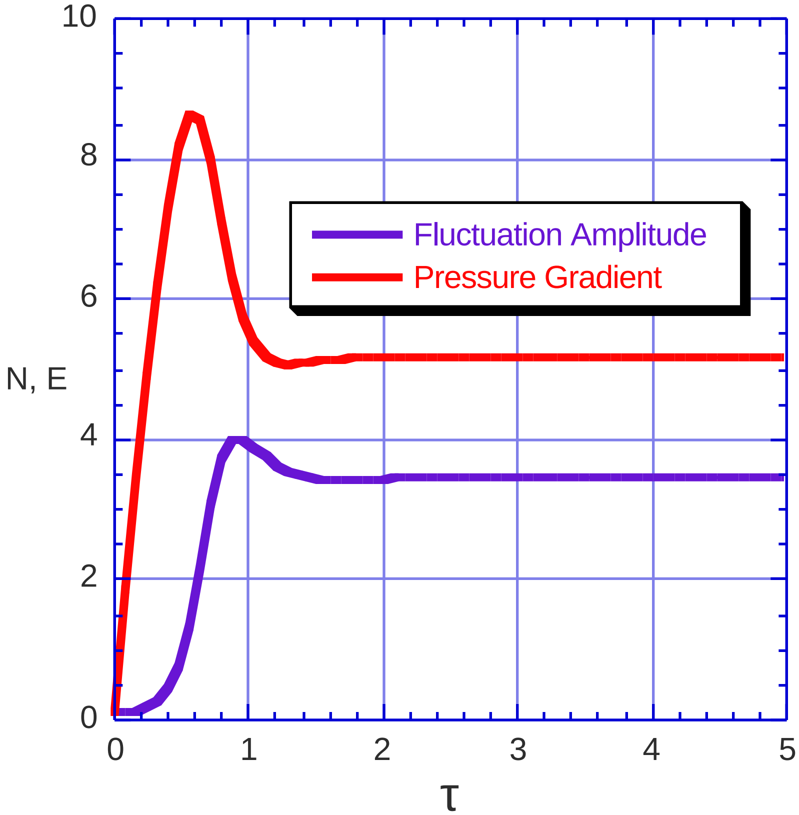
k_1: 1	k_2: 2
k_3: 7	Q: 10

Input Parameters:

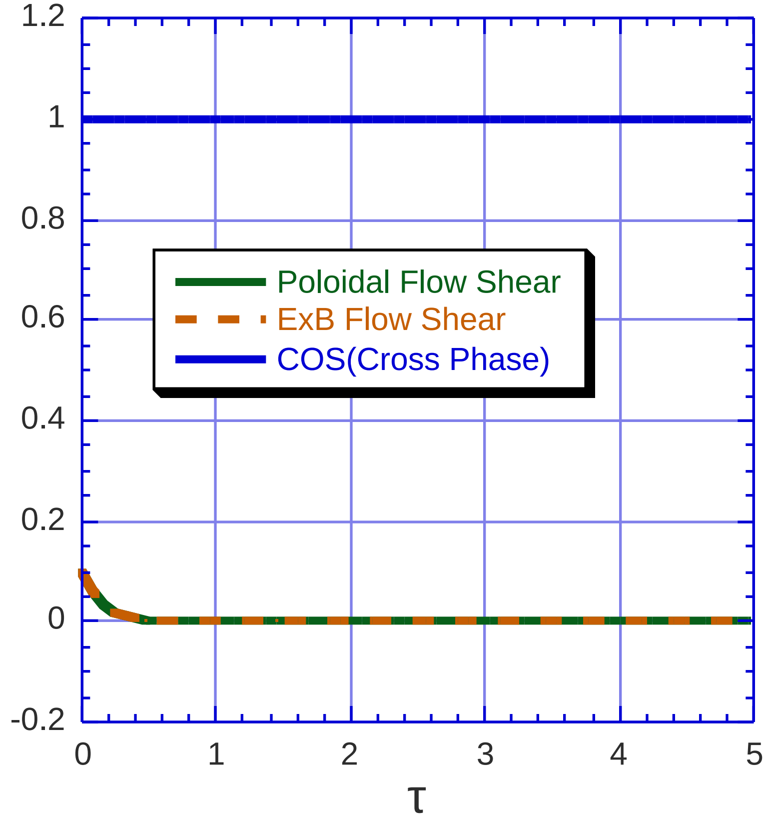
k_1: 1	k_2: 2
k_3: 7	Q: 24

An “L-mode” fixed point without cross phase evolution...

Subcritical forcing / Fixed cross phase



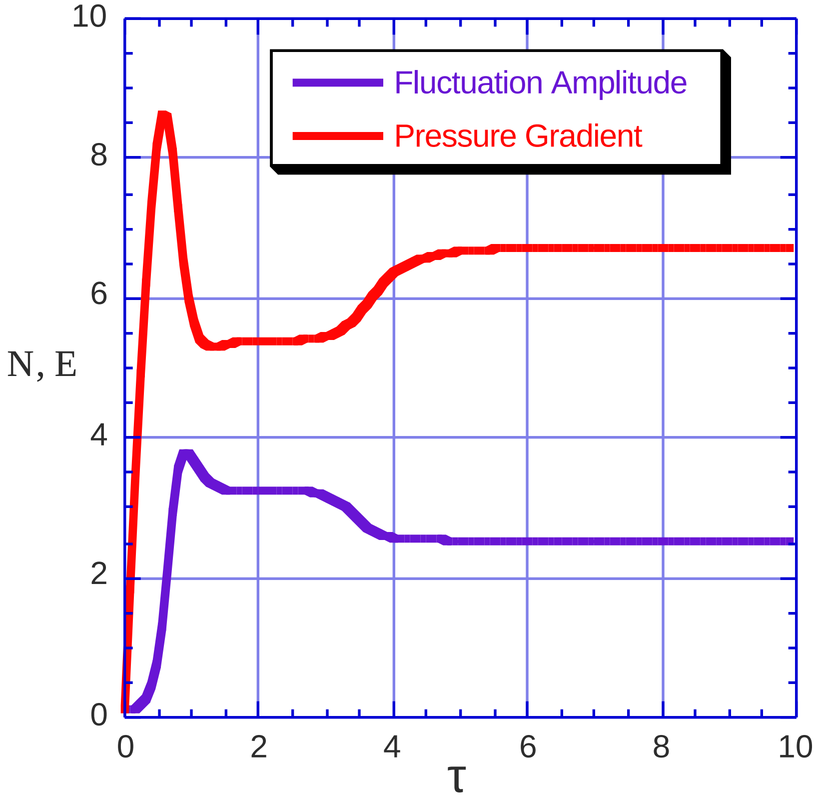
Subcritical forcing / Fixed cross phase



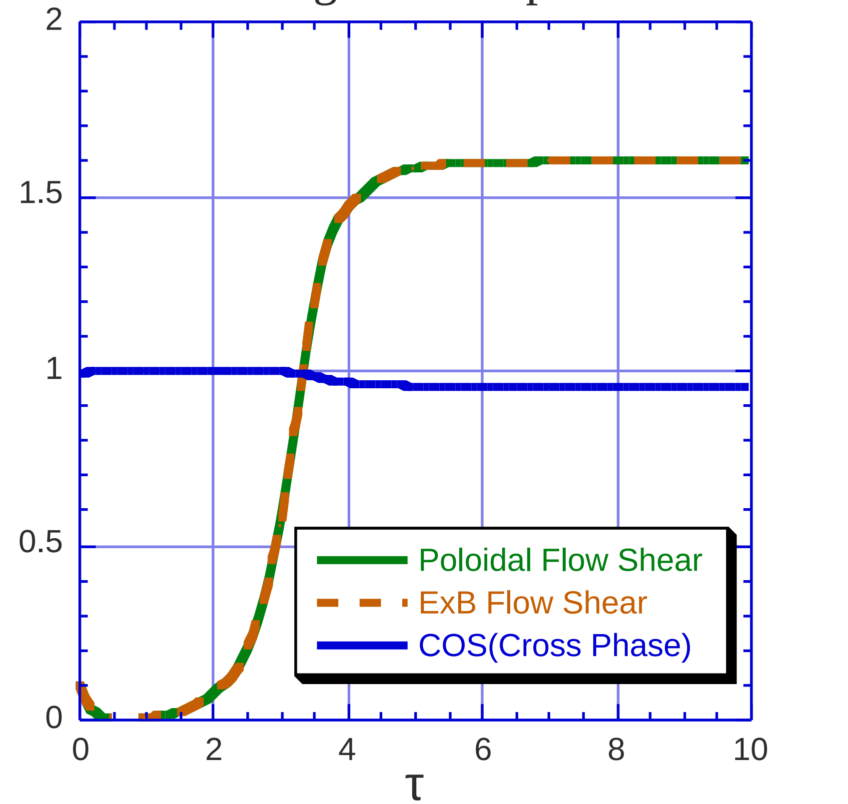
k1=1.5; k2=2; k3=7; Q=23

...becomes an "H-mode" fixed point with cross phase evolution.

Subcritical forcing / Cross phase evolution



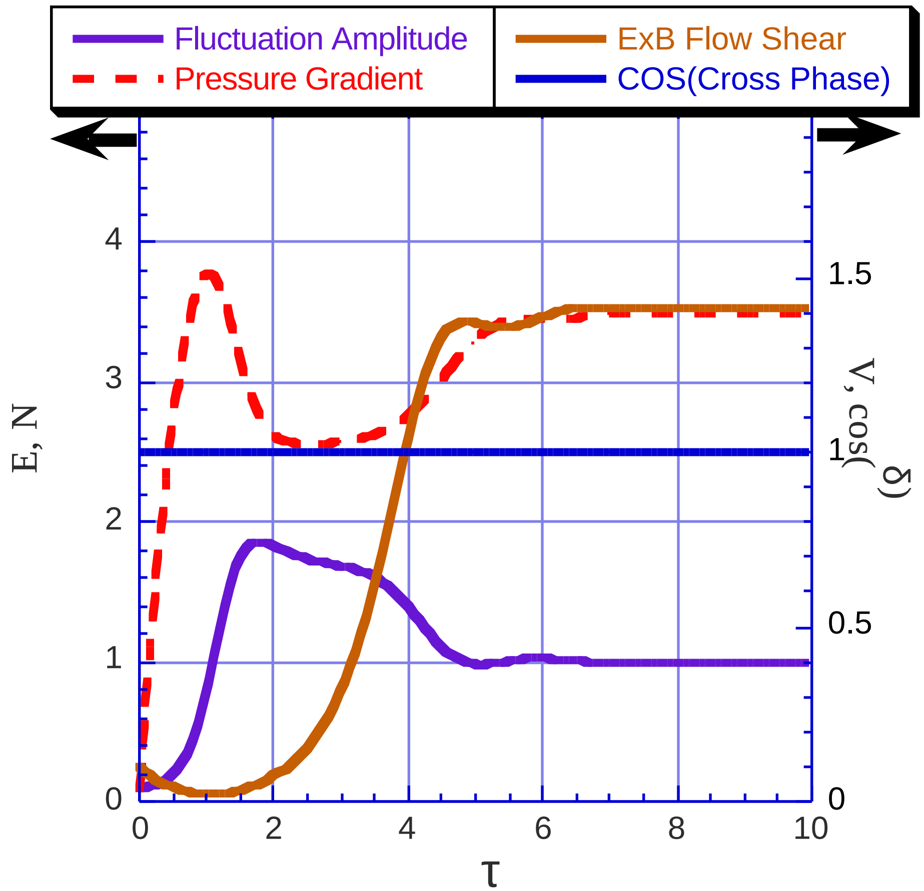
Subcritical forcing / Cross phase evolution



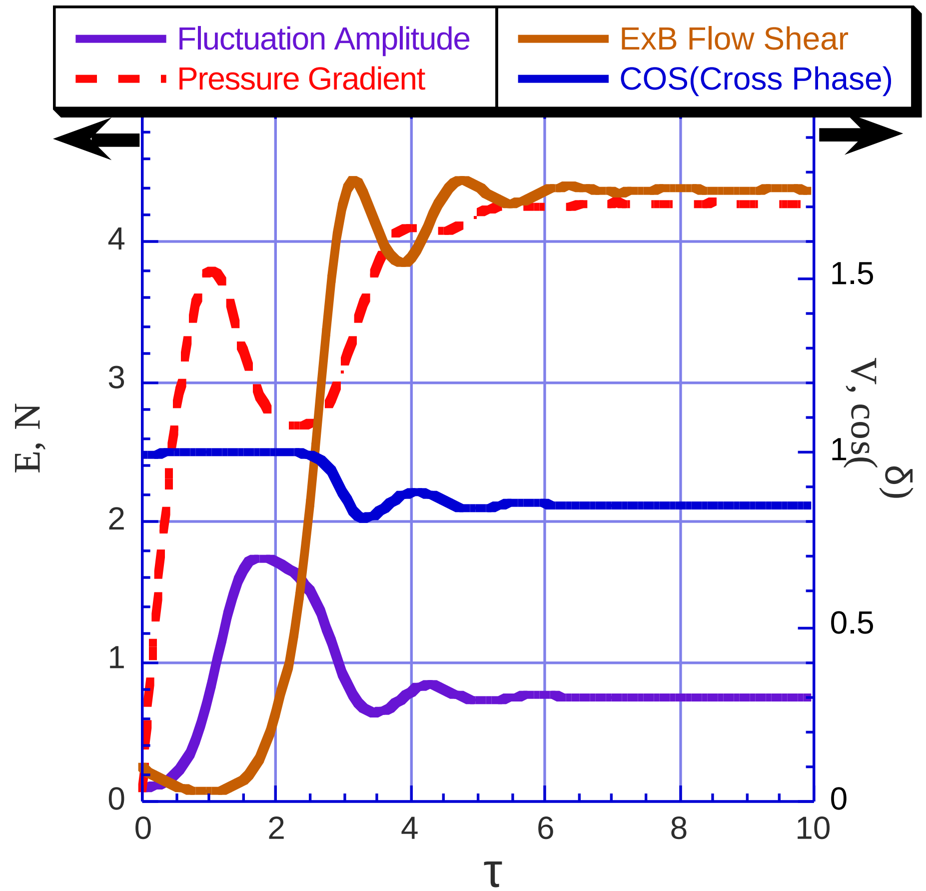
k8=0.1; k9=0.4; k10=0; k11=1; k12=1; k13=0.1; k14=1

Cross phase effects can also modify the timing of the transition.

Supercritical forcing / Fixed cross phase



Supercritical forcing / Cross phase evol.



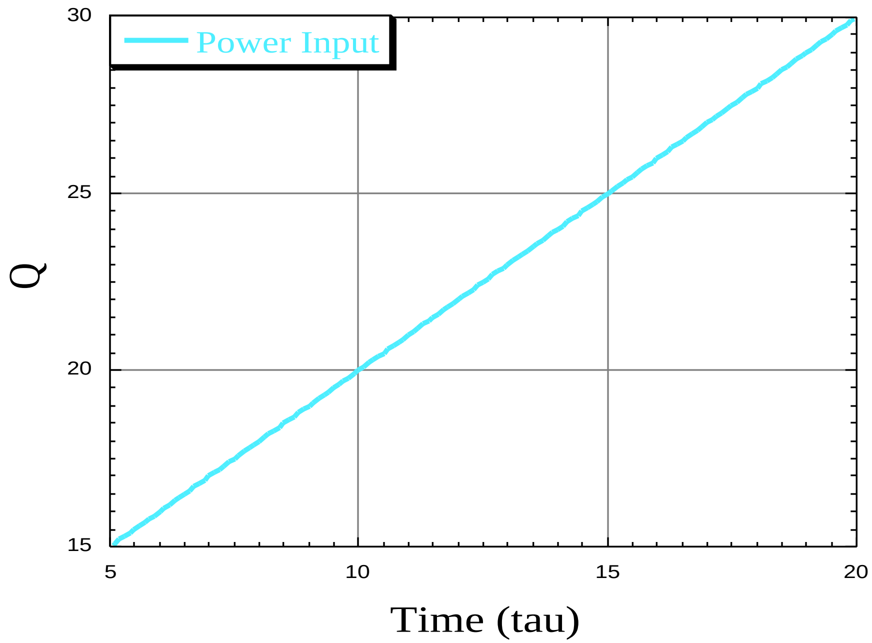
Phase transitions between the two fixed points can be triggered by increasing the power

Linearly Varying

Power Input:

$$Q = Q_0(1 + \frac{\tau}{\tau_0})$$

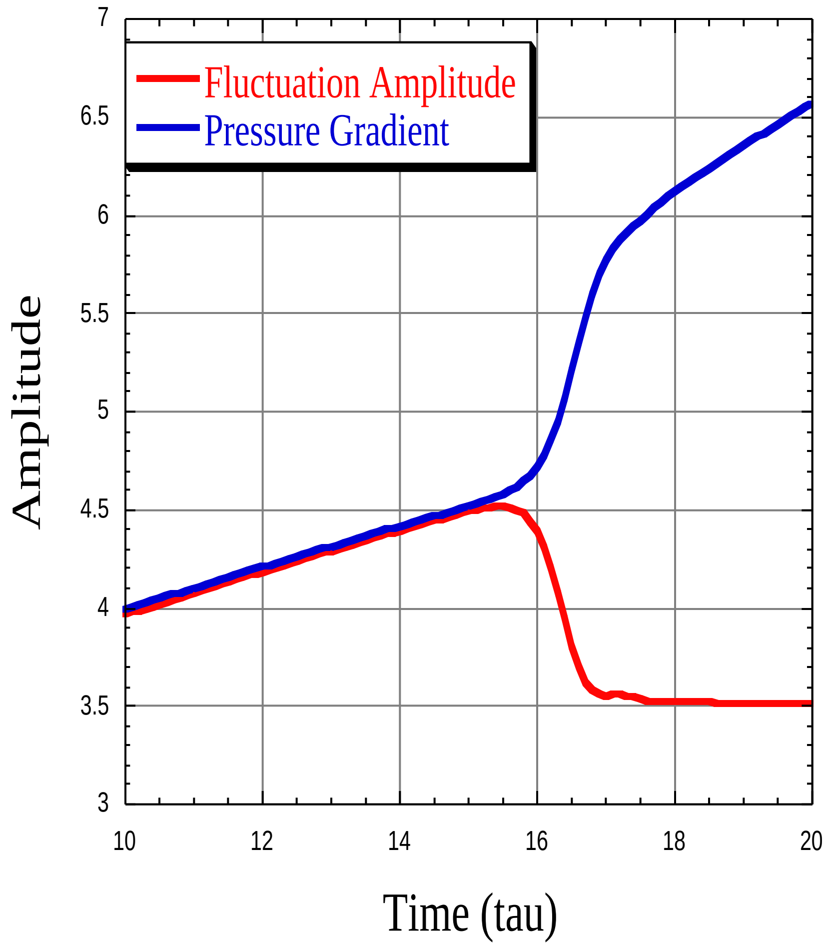
(Same for all cases below)

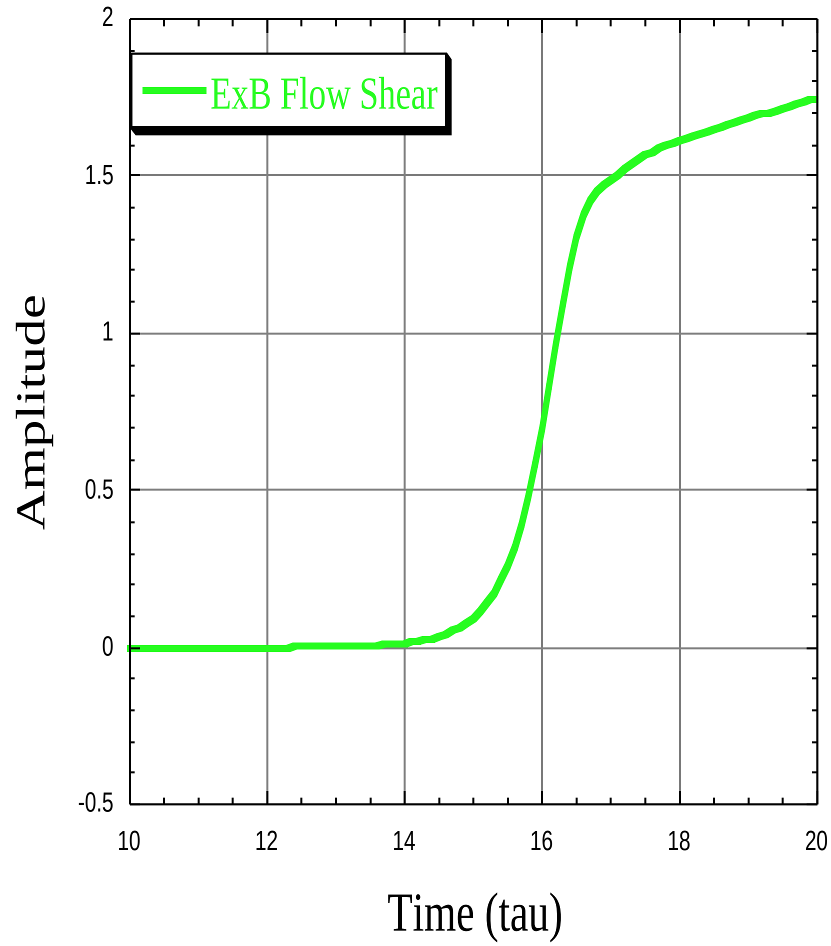


Input Parameters:

k_1: 1 k_2: 2 k_3: 7 k_4: 10 k_5: 2 k_6: 1 k_7: 3 $Q(0)$: 10

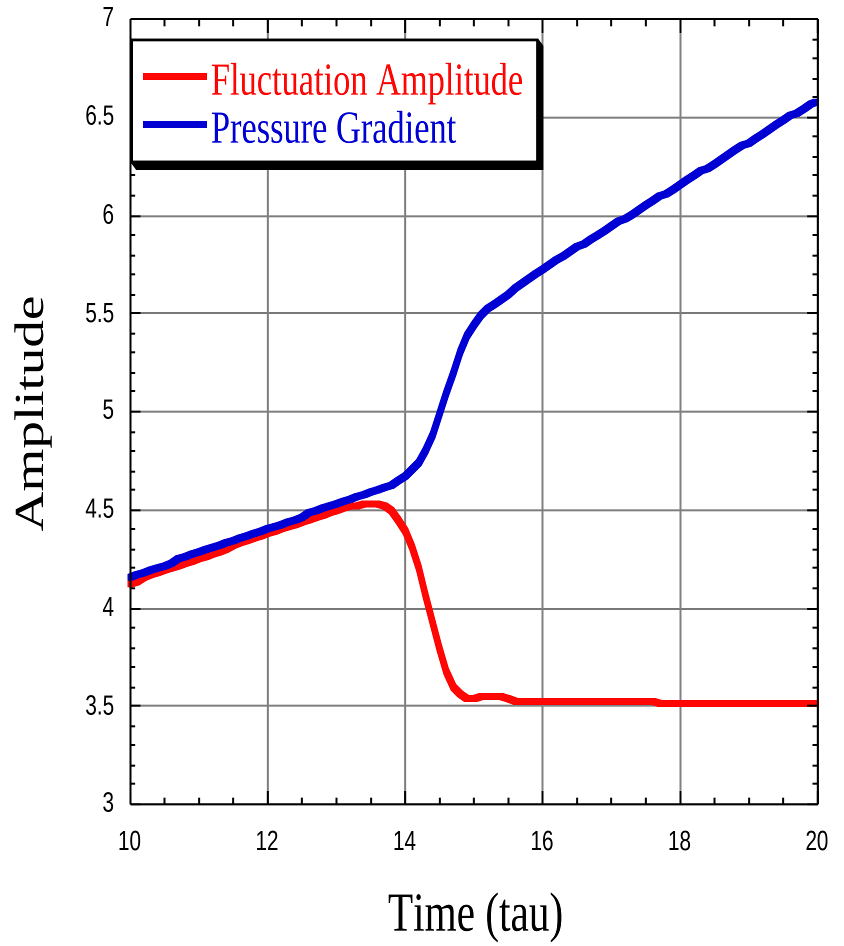
Predator-Prey Model without Cross Phase

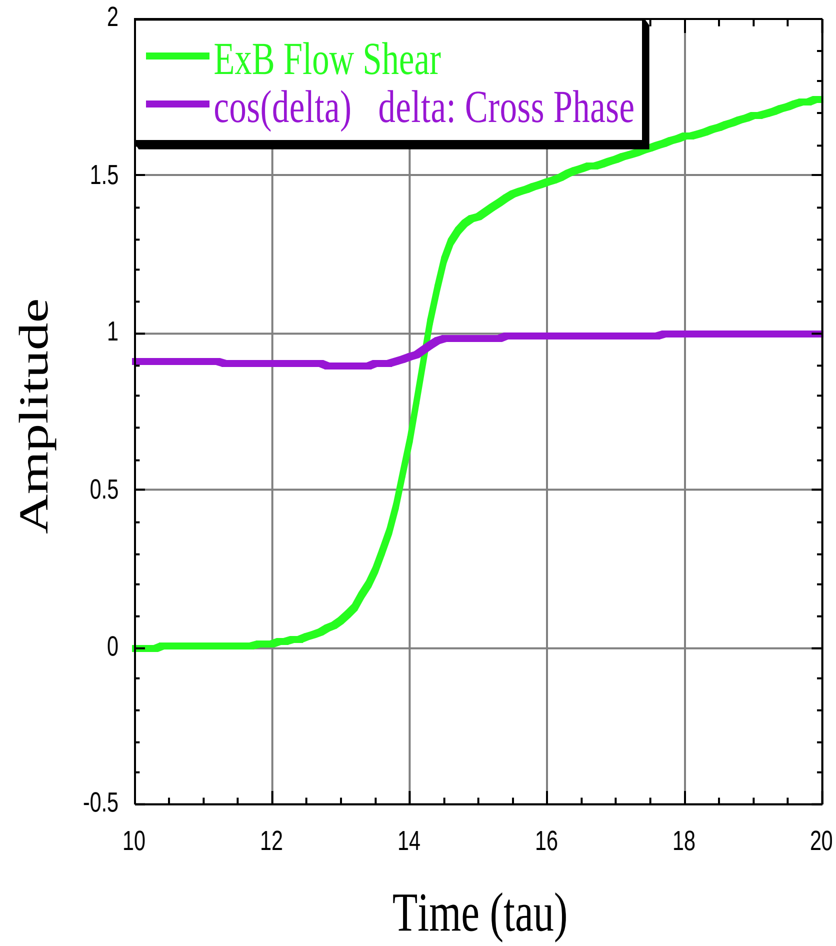




Generation of flow at $\tau \approx 14$ increases pressure gradient and suppresses fluctuation amplitude. Power threshold $Q \approx 24$.

Predator-Prey Model with Cross Phase #1





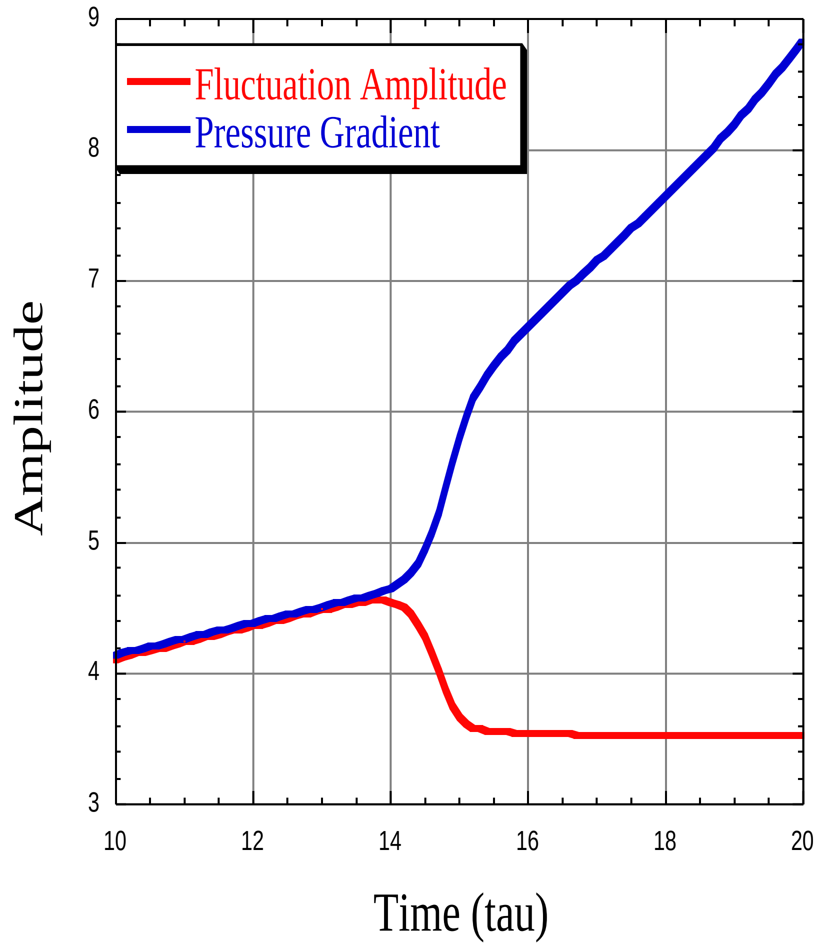
Generation of flow at $\tau \approx 12$ increases pressure gradient and suppresses fluctuation amplitude. Power threshold $Q \approx 22$.

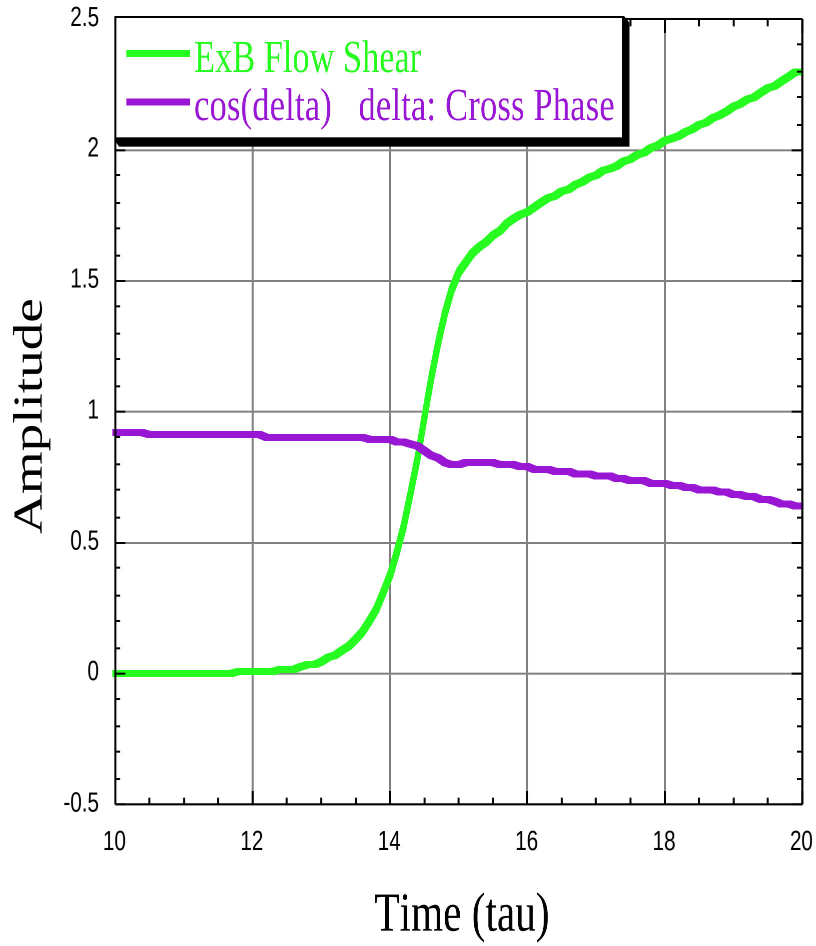
If the cross phase is more sensitive to nonlinear modifications, the density gradient increases much more dramatically after the transition

Input Parameters:

k_1: 1 k_2: 2 k_3: 7 k_4: 10 k_5: 2 k_6: 3 k_7: 3 $Q(0)$: 10

Predator-Prey Model with Cross Phase #2





Generation of flow at $\tau \approx 12.5$ increases pressure gradient and suppresses fluctuation amplitude. Power threshold $Q \approx 22.5$.

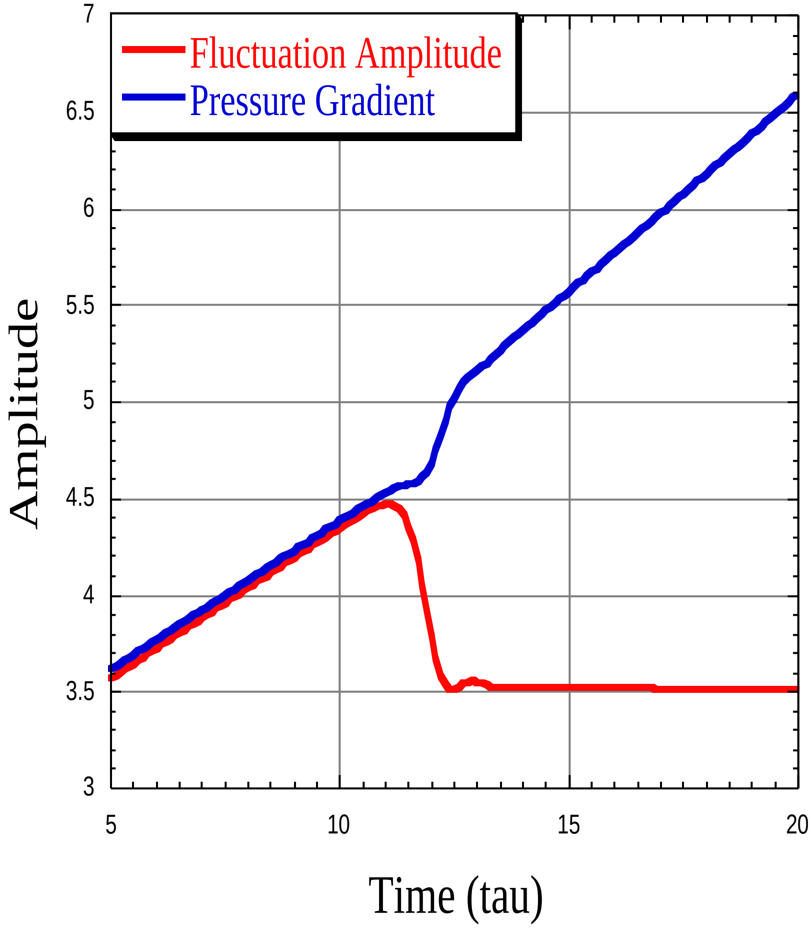
If the cross phase is more easily modified, the power threshold for the transition is lower

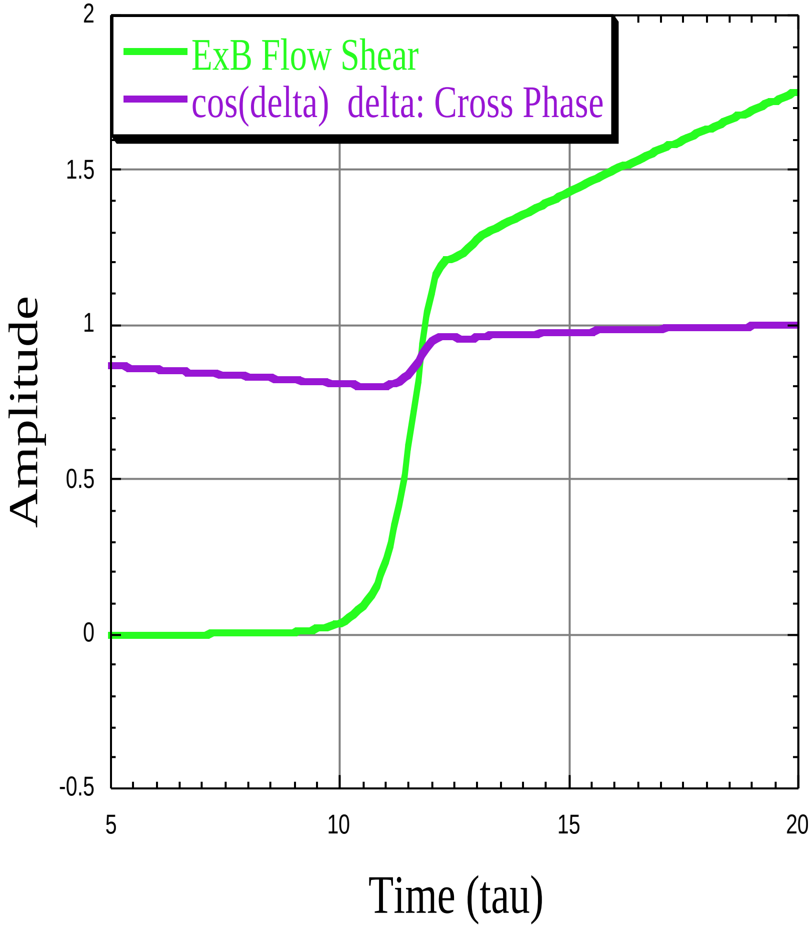
Input Parameters:

k_1: 1 k_2: 2 k_3: 7 k_4: 7 k_5: 2 k_6: 1 k_7: 3 $Q(0)$: 10

- The weight on cross phase in cross phase evolution is increased.

Predator-Prey Model with Cross Phase #3





Generation of flow at $\tau \approx 9$ increases pressure gradient and suppresses fluctuation amplitude. Power threshold $Q \approx 19$.

V. Future work

- Look at solutions to the coupled system including cross phase dependence in both flow and density gradient evolution
- Long term possibility is to include cross phase between $\tilde{T}$ and $\tilde{v}_r$ as well
- Derive a better cross phase evolution equation, e.g.

$$\frac{d\delta}{d\tau} = -\left(\cos\delta - 1 + k_6 E + k_7 U^2\right)^2 k_4 N$$