Lecture 4

Measuring the time interval between events.

1. Coordinate time
2. Proper time
3. Spacetime interval

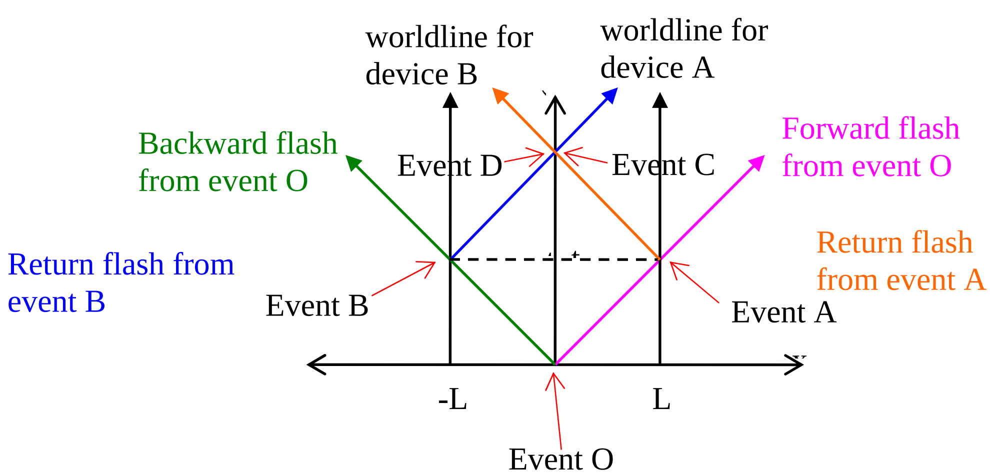
Coordinate Time Interval

Within a given inertial reference frame we now know how to synchronize clocks and so can read the spacetime coordinates of events. We can also use the radar method of finding the spacetime coordinates for the events if we so choose and we know that we will get the same results.

Find the time between two events. Construct a device that will set off a flash bulb when it is illuminated by a light flash. Place one such device (call it A) at L seconds in the positive x direction from the master clock and a second such device (call it B) at L seconds in the negative x direction from the master clock. Let the following sequence of events take place.

- Event O When the master clock reads t_O set off a flash bulb at the origin.
- Event A The light from event O reaches A and sets off the flash bulb at A.
- Event B The light from event O reaches B and sets off the flash bulb at B.
- Event C The light from event A reaches the master clock.
- Event D The light from event B reaches the master clock.

Draw these events on a spacetime diagram and include the worldliness for the two flash devices, for the outgoing light flashes, and the returning light flashes.



Consider the triangle defined by the light flash from O to A, the light flash from A to C and the worldline of the master clock from O to C. The light flashes have slopes of +1 and then -1. That means that the angle at A is a 90° angle. The altitude of that triangle from the side OC to the vertex at A is L.

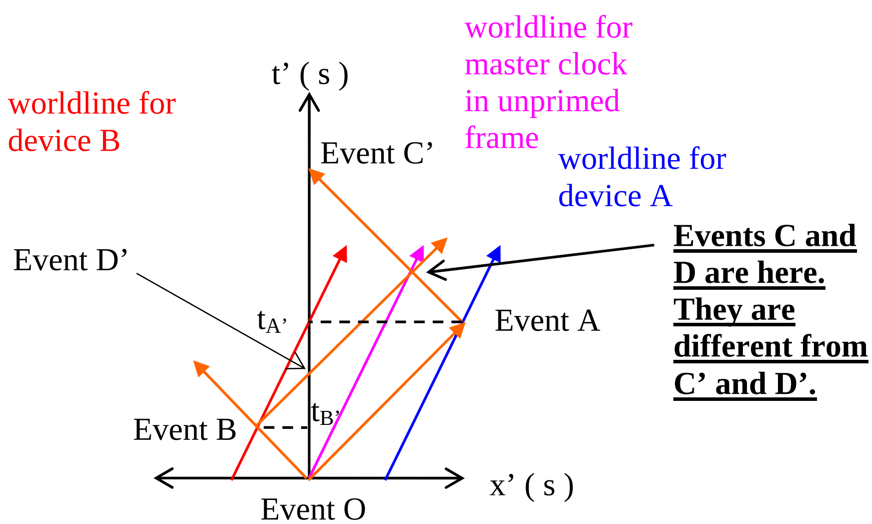
Now consider the comparable triangle OBD. It is also a right triangle with a corresponding altitude of L. Thus the two triangles are congruent and OC = OD.

Using the radar method of determining spacetime coordinates we find that

$$t_A = t_B = \frac{t_C + t_O}{2};$$

events A and B are simultaneous.

Now view these same events from a primed reference frame moving at a velocity of 0.5 to the left with respect to the above unprimed reference frame. In the primed reference frame all of the bits of the unprimed frame move in the positive x' direction. Let us take the origins to coincide at t_O.



While triangles OAC' and OBD' are still both right triangles, it is clear that they are not congruent and that $t'_A > t'_B$. The events are not simultaneous. Be sure to draw the world line for the master clock as that is how you locate events C and D. They must still be simultaneous as they are not separated in space. The purpose of C' and D' is to compute $t_{A'}$ and $t_{B'}$. We do not do this because of the Lorentz contraction of the distance between the master clock and the two devices. The point here is simply to see that the events are no longer simultaneous.

The time interval between events is reference frame dependent. Refer to this way of finding the time interval as the **coordinate time interval**.

Think about what this means. In the unprimed reference frame I have a bunch of synchronized clocks. The clocks present at events A and B each spit out a ticket the reads “The time of this event is $t_O + L$”. Anybody in the entire universe can look at those tickets and see that they say exactly the same thing.

How can the observers in the primed reference frame then say that event B happened before event A? For them this is easy. They have their own set of clocks. The clock present at event B spits out a ticket that reads “The time of this event is $t_O + \frac{L}{\sqrt{3}}$” and the clock that is present at event A spits out a ticket the reads “The time of this event is $t_O + L \cdot \sqrt{3}$”. What do the observers in the primed reference frame say when the unprimed observers hold up their tickets? They say “your clocks are not synchronized so naturally the numbers on the tickets are not correct.” The observers in the unprimed reference frame say exactly the same thing.

Both are correct! Clocks synchronized in one inertial reference frame are not synchronized in a different inertial reference frame. This is a necessary consequence of a universal speed for light.

Coordinate time intervals are frame dependent.

Make analogy with translated vs. rotated reference frames.
- Consider a pair of locations that in a particular reference frame have the same x coordinate.
- Now examine them from any translated reference frame. The x coordinates might change but the pair would always change in the same way so that their difference is always zero.
- We might grow to think of the x coordinates as very special in that for any two locations their difference must always be some fixed value.
- Now consider a frame rotated relative to the original frame. The x coordinates will change by differing amounts depending upon their y coordinates.
- This might strike us as very strange if we had never seen rotated reference frames before.

Rotating through an appreciable angle is analogous to shifting to another inertial reference frame that is moving at an appreciable fraction of the speed of light.

Proper Time Interval

The issue in the measurement of coordinate time intervals arises because observers in different reference frames cannot agree upon what set of clocks, spread through space, are synchronized. Suppose we eliminate this problem by using only 1 clock. There will be no issue of synchronization to worry or disagree about.

Take a clock and have it be present at event A. It spits out a ticket with the time of event A printed on it. Now move it to the location of event B and wait for B to occur. When it does push the button and the clock spits out a time ticket for event B. The difference between the two printed times is then the time interval between events A and B. This time difference is known as the **Proper Time Interval**. Proper actually comes from a translation of a German word and might better be called proprietary time – it belongs to that particular clock.

There is a problem with proper time intervals though. It depends upon how you move the clock from the location of event A to the location of event B. We do not just now have the machinery to compute the nature of that dependence but it can be done. In October 1971 Hafele and Keating procured two highly accurate cesium clocks (good to about 2ns per day) and synchronized them at an airfield. One was flown westward around the world and returned to the airfield while the other stayed in place. The prediction was that the traveling clock should gain 275ns relative to the non-traveling clock. The actual result was 273±7ns.

Every observer agrees as to the proper time interval indicated by any given clock but different clocks do not agree.

Spacetime Interval

There is one possibility to join proper time intervals and coordinate time intervals. The clock present at both events could be an inertial clock. If you measure the time interval in this way you get a unique quantity that we call the spacetime interval.

The coordinate time interval is rather like a difference in a single coordinate when moving between two points.

The proper time interval is something like measuring the path length between two points.

The spacetime interval is **something like** the Euclidian distance between the two points. We will need to generate a new expression to tell us the spacetime interval.