# Physics 122
# Chapter 26
# Problem Solutions

There are two expressions you need to be able to use.

The metric equation $(\Delta t_1)^2 - (\Delta d_1/c)^2 = (\Delta s)^2 = (\Delta t_2)^2 - (\Delta d_2/c)^2$

and the definition of gamma $\gamma = \dfrac{1}{\sqrt{1-\left(\dfrac{v^2}{c^2}\right)}}$.

Q3 The ball will land on the roof of the railroad car (ignoring air resistance and assuming that the velocity of the train is constant). Both the ball and the car are already moving forward, so when the ball is thrown straight up into the air with respect to the car, it will continue to move forward at the same rate as the car and fall back down to land on the roof.

Q5 If you were in a spaceship traveling at 0.5*c* away from a star, its starlight would pass you at a speed of *c*. The speed of light is a constant in any reference frame, according to the 2nd postulate of special relativity.

Q7 Time actually passes more slowly in the moving reference frame. It is not just that it seems this way, it has actually been measured to pass more slowly, as predicted by special relativity.

Q9 You would not notice a change in your own heartbeat, mass, height or waistline. No matter how fast you are moving relative to Earth, you are at rest in your own reference frame. Thus, you would not notice any changes in your own characteristics. To observers on Earth, you are moving away at 0.5*c*, which gives $\gamma = 0.87$. To these observers, it would appear that your heartbeat has slowed by a factor of 0.87, that your mass has increased by a factor of 1/0.87 = 1.15, and that your waistline has decreased by a factor of 0.87 (all due to the relativity equations for time dilation, mass increase, and length contraction), but that your height would be unchanged (since there is no relative motion between you and Earth in that direction).

P1 Here you can use your mantra, "Moving meter sticks are short by a factor of gamma." Compute $\gamma$. It is 1.51. The moving (short) length is 28.2m so the at rest length is (1.51)(28.2m) = 42.6m.

P4 In this problem you are given the at rest length and asked to find the shortened (moving) length. $\gamma$ is 1.81 The distance as measured by the people in the spaceship would be 69.1 light years.

P8 Remember that the relative velocity will be the same no matter who measures it. So $\gamma = 0.90$. Solve for v. I get 0.436 times the speed of light.

P11 We will ignore any details of exactly how the ship was moving; they matter but we don't have them or the ability to work with them. However, if the trip started and ended from the spaceport on the Earth, then the Earth and the ship are not equivalent. We can think of the Earth as being at rest but the ship had to be doing some accelerating. We must apply our other mantra "Moving clocks tick slowly by a factor of gamma." from the perspective of the Earth. Compute $\gamma$. It is 1.84. The elapsed time on the ship will be less than that on the Earth regardless of who is asked.

If the ship's clocks read 5 years then the Earth's clocks will read 9.2 years. If the Earth's clocks read 5 years then the ship's clocks will read 2.7 years.

P14 Here the metric equation works well. In the reference frame of the pion, the distance between creation and decay is zero. The time between creation and decay is given as $2.6\cdot10^{-8}$s. We want the distance in the laboratory between creation and decay to be 15m. Compute the time interval in the laboratory and divide the distance by the time to get the speed.

$$\left(\Delta t_{pion}\right)^2 - \left(\frac{\Delta x_{pion}}{c}\right)^2 = \left(\Delta t_{lab}\right)^2 - \left(\frac{\Delta x_{lab}}{c}\right)^2$$

$$\left(2.6\cdot10^{-8}s\right)^2 - \left(\frac{0}{c}\right)^2 = \left(\Delta t_{lab}\right)^2 - \left(\frac{15m}{3\cdot10^{8}m/s}\right)^2$$

$$\Delta t_{lab} = 5.64\cdot10^{-8}s$$

$$v = \frac{15m}{5.64\cdot10^{-8}s} = 2.66\cdot10^{-8}s = 0.887c$$

Extra Problem #1

The spaceship is moving at a constant velocity relative to an inertial reference frame ( the solar system ) and it is therefore an inertial frame as well. Let Event A be the entrance of the ship into the solar system and let Event B be its departure. The clock on the ship is present at both events so it reads a proper time between them. Because it is an inertial clock, the time it reads is also the spacetime interval.

The clocks in the solar system give the coordinate time as they are attached to an inertial reference frame. With the coordinate distance between the events, this allows us to find the spacetime interval ( and thus the time interval measured by the ship's clock ) by using the metric equation.

$$\begin{aligned}\Delta s^2 &= \Delta t^2 - \Delta d^2 \\ &= (13.2 hours)^2 - (10.5 hours)^2 \\ &= 64 hours^2 \\ \Delta s &= 8 hours\end{aligned}$$

Extra Problem #2

Part a.

If the muon's internal clock agreed with clock's on the Earth, how far would it go before it decayed?

Just multiply speed and time.

$$0.998c \cdot 2\mu s = 600m$$

Part b.

Using your understanding of relativistic time, find out how far if really travels as seen by an observer on the ground. Here we are interested in finding the coordinate distance, Δd, and we know the spacetime interval, Δs. How do you know the spacetime interval? The muon is a clock! It is present at its creation and at its decay. It is moving with a constant velocity with respect to the ground which we take to be an inertial reference frame. But is this enough? If we try to use the metric equation to find Δd, we will still need Δt. Is there a connection between those two? ( We call it speed, v. )

$$\Delta s^2 = \Delta t^2 - \frac{\Delta d^2}{c^2}$$

$$\Delta t = \frac{\Delta d}{v}$$

$$\Delta s = \Delta d\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}$$

$$\Delta d = \frac{\Delta s}{\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}}$$

$$\Delta d = \frac{2\mu s}{\sqrt{\frac{1}{(.998c)^2} - \frac{1}{c^2}}}$$

$$\Delta d = 9,480m$$