# Physics 122
# Chapter 24
# Problem Solutions

Q5 The locations of the interference maxima on a screen a distance L from the slits were given by

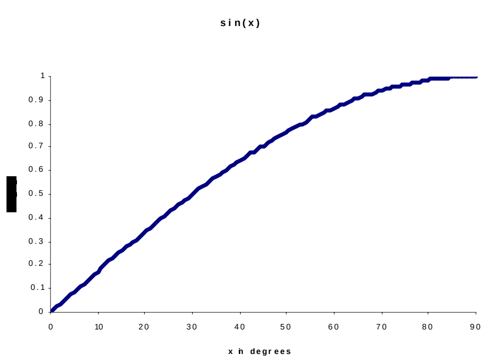
$$y_m = \frac{m \cdot \lambda \cdot L}{d} \qquad \text{(Q5-1)}$$

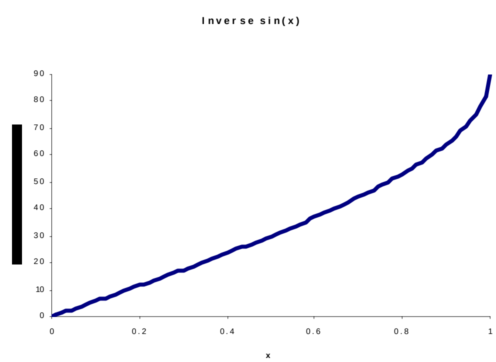
The effect of putting the apparatus into a tank of water is to reduce the wavelength of the light by a factor of the index of refraction of water. As can be seen from (Q5-1), this would reduce the distance from the center of the pattern to any of the maxima; the pattern contracts toward its center.

Q6 The angular location of the first interference minimum is given by

$$\theta = \sin^{-1}\left(\frac{\lambda}{D}\right) \qquad \text{(Q6-1)}$$

where D is the width of the slit. Other minima are similarly related to the wavelength. To see the effect changing the wavelength will have on θ you need to know how inverse sine behaves. If I restrict the angle to 0º → 90º then $\sin^{-1}(x)$ is a function. Here is a graph of sine and its inverse. The crucial point here is that when the argument of the $\sin^{-1}$ function gets smaller so does the value of the function.




Thus, when the wavelength is reduced, the interference pattern contracts toward the center.

Q8 The interference pattern can only be explained by waves. Not only do you get light in regions where you expect darkness but you also have places where the addition of the second slit *reduces* the intensity of the light on the screen. One might be able to make up some sort of particle model that has the light particles bouncing off the sides of the slit thereby spreading the light across the screen. However, there is no provision of any sort in the particle model of light that allows an addition of light to reduce the intensity of light that was already present.

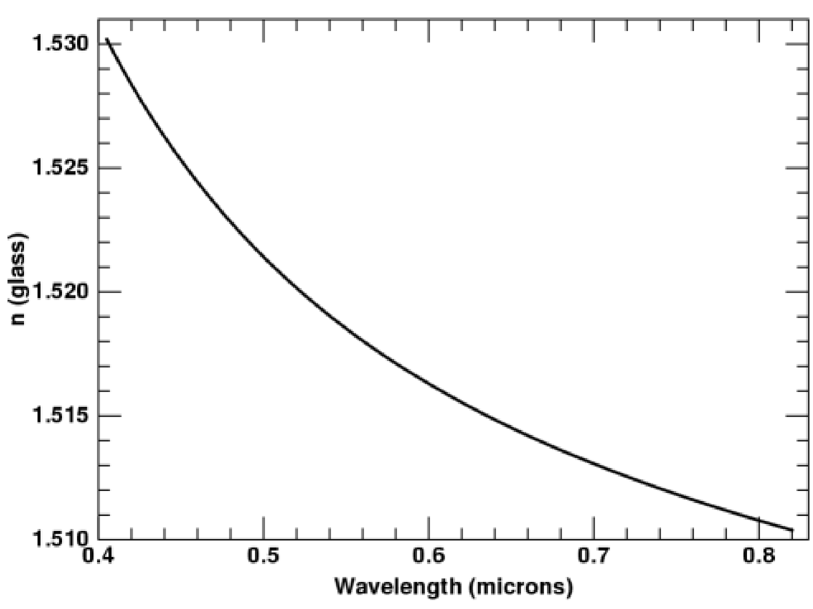
Q10 In the derivation of the interference pattern from a pair of slits, it was assumed that the very same train of waves passed through the two slits. The headlights are generating independent waves and so there will be no interference pattern of any sort.

Q13 The central idea governing how light behaves as it passes through a lens is Snell's law. Snell's law appeals to the index of refraction of the light to help determine how much the light bends at each surface.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Suppose that we let one of the media be air so that its index of refraction is 1.000. It follows from looking at the graph of the sine function ( see Q6 ) that for any given value of θ on the air side of the surface, as the index of refraction for the lens grows larger, the angle inside the lens will grow smaller. A larger index of refraction for the lens means more bending of the light. This result is independent of the nature of the lens or whether the light is entering or exiting the lens.

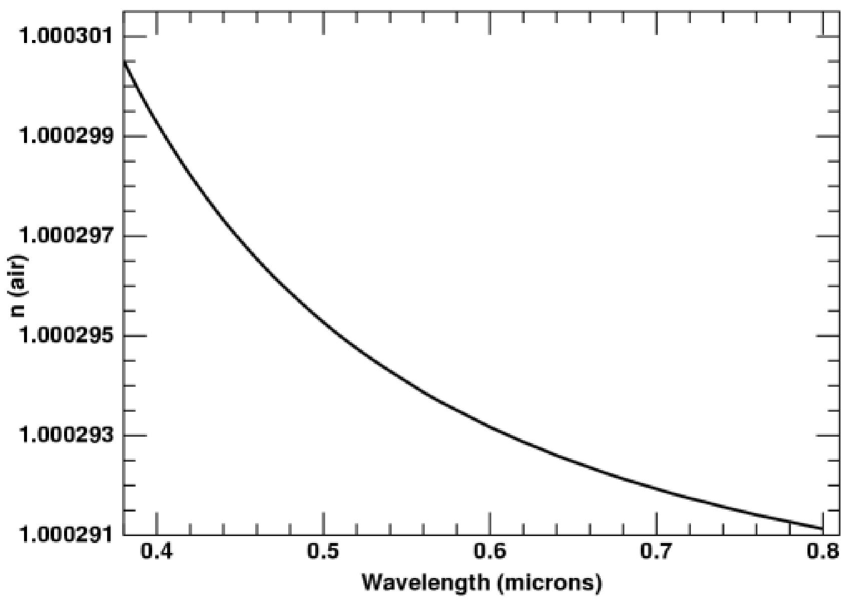
With this in mind look at this figure or its equivalent in your book.




Refractive index of glass

The short wavelengths ( blue ) have a larger index of refraction than do the long wavelengths ( red ). Because the blue light will bend more regardless of the type of lens, the focal lengths will be smaller for blue light than for red for both converging as well as diverging lenses.

Why do we not see color separation in air? If you look through the hot air rising from a heating element, objects on the far side wave about. If the air is bending the light why do we not see color here too? The graph corresponding to the one for glass gives the answer. Pay attention to the vertical scales for each graph.



Refractive index of air

All visible light travels at essentially the same speed in air.

Q20 In general, red light slows down less and thus refracts less than does blue light as it passes through a prism. See the first graph in Q13. Thus red light would be on the top of the spectrum for a prism. For a diffraction grating, the construction for constructive interference required the additional path length to be a multiple of the wavelength of light. Longer wavelengths pushed the pair of paths farther to the bottom. See (Q5-1). Thus blue light, having the shortest wavelength, will be found nearest the center of the pattern; that is, blue will be on the top of the spectrum.

Q26 If the thickness of the oil film is very small compared to the wavelength of the light then any phase shift of the light reflected from the first surface relative to the light reflected from the second surface must be due to the reflections themselves and not to any extra path length of second path. So either both must suffer a phase shift or neither suffers a phase shift if they are to remain in phase and produce a bright fringe. Assume that the index of air is 1.0 and that of water is 4/3. Either the index of refraction for the oil is less than that of air and greater than that of water ( this would give no phase shift at each reflection ) or the index of refraction of the oil must be greater than that of air and less than that of water ( this would give a phase shift for each reflection ). The former is not possible and so we conclude that $n_{air} < n_{oil} < n_{water}$.

P13 If we look at the paths that light follows in getting to the screen ( through the top and bottom slits top ) we see that they are the same except for the little bit just to the left of the slits where the top path goes through the film and the bottom path goes through air. Do the standard thing! Count up the number of wavelengths that fit in the film ( $N_{top}$ ) and the number that fit in the same thickness of air ( $N_{bottom}$ ).

$$N_{top} = \frac{\text{film thickness}}{\text{wavelength in film}}$$

$$N_{bottom} = \frac{\text{film thickness}}{\text{wavelength in air}}$$

Let t be the film thickness and n be the index of refraction for the film. We are supposed to have destructive interference so the difference in these two numbers should be m + ½.

$$N_{top} - N_{bottom} = m + \frac{1}{2}$$

$$\frac{t}{\lambda/n} - \frac{t}{\lambda} = m + \frac{1}{2}$$

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n-1}$$

We are to find the thinnest film so we take m = 0. I get t = 533nm.

P15 Just apply Snell's law to each wavelength and subtract the angles of the refracted beams. The angles are 35.76° and 35.98°. The difference is 0.22°.

P25 If you are stuck you should review Q6.

Solve (Q6-1) for the slit size with the known wavelength and spot size.

$$D = \frac{\lambda_1}{\sin(\theta_1)} \tag{P25-1}$$

Because these angles are very small,

$$\sin(\theta) \simeq \tan(\theta) \tag{P25-2}$$

So, using half the width of the central spot and the distance to the screen to get

$$\tan(\theta) = \frac{w_1 / 2}{L} \tag{P25-3}$$

we rewrite (P25-1) as

$$\begin{aligned} D &= \frac{L\lambda_1}{w_1 / 2} \\ &= \frac{2L\lambda_1}{w_1} \end{aligned}$$

where L is the distance to the screen and $w_1$ is the width of the central spot.

Our goal is to find the spot size given the second wavelength and the same slit width. The spot size is contained in (P25-3) and we will need a version of (P25-1).

$$\begin{aligned} \sin(\theta_2) &= \frac{\lambda_2}{D} \\ \tan(\theta_2) &= \frac{w_2 / 2}{L} \end{aligned}$$

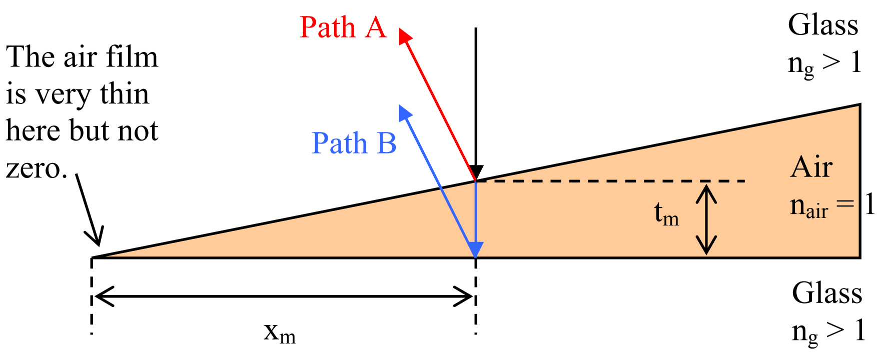
Using our approximation, (P25-2), and the calculated value for the slit width D we have,

$$\begin{aligned} \frac{w_2}{2L} &= \frac{\lambda_2}{D} \\ w_2 &= \frac{2L\lambda_2}{D} \\ w_2 &= w_1 \frac{\lambda_2}{\lambda_1} \end{aligned}$$

Thus the new central spot width is 2.6cm.

P37 Since this is the first order diffraction line, $d \cdot \sin(\theta) = \lambda$. The number of lines per meter is 1/d; this is $5.79 \cdot 10^5$ lines per meter.

P44 A key to understanding this problem is realizing that the thin film is the air between the sheets of glass. Each sheet of glass is so thick compared to the wavelength of light that there are no interference effects due to the individual sheets of glass. Focus on the film of air.




Analyze the situation for the general location indicated in the figure. Proceeding in the standard way we have

$$N_A = \frac{0}{\lambda} + 0$$

and

$$N_B = \frac{2t}{\lambda} + \frac{1}{2}.$$

The problem statement refers to dark fringes so we are talking about destructive interference here and we write

$$N_B - N_A = m + \frac{1}{2}$$

$$\left[\frac{2t_m}{\lambda} + \frac{1}{2}\right] - [0] = m + \frac{1}{2}$$

$$t_m = \frac{m \cdot \lambda}{2}$$

These are the thicknesses of the air film where we see dark lines. The first one is on the left where the thickness of the film is essentially zero. For this location m = 0. If this is the 1st dark line then the 28th dark line corresponds to m = 27. The foil is thus 13.5 wavelengths or 9.05μm thick.

P62 In part a) we need the locations of the interference maxima for a pair of slits. We will use the usual small angle approximation

$$y_m = \frac{mL\lambda}{d}$$

So, adjacent maxima are separated by

$$\begin{aligned}\Delta y_m &= y_{m+1} - y_m \\ &= \frac{(m+1)L\lambda}{d} - \frac{mL\lambda}{d} \\ &= \frac{L\lambda}{d}\end{aligned}$$

Given that this is 2.0cm, we can solve for the slit spacing. It turns out to be 0.10mm.

In part b) we are to find a second wavelength given a relationship between the locations of interference minima for each wavelength. Now we want the location for destructive interference.

$$y_m = \frac{(m+1/2)L\lambda}{d}$$

We know that the fourth order minimum for 500nm light occurs at the same point as the fifth order minimum for the new wavelength. Set the locations equal and solve for the new wavelength.

$$\begin{aligned}\frac{(5+1/2)L\lambda}{d} &= \frac{(4+1/2)L\cdot 500nm}{d} \\ \lambda &= \frac{9}{11}500nm \\ \lambda &= 410nm\end{aligned}$$

Notice that neither the slit spacing nor the distance to the screen was important to this result. If this seems strange, picture the construction for the two slit experiment. Regardless of the wavelength, the path from the top slit to a given point on the screen is what it is. The same is true for the path from the lower slit. The difference in the two paths is a fixed distance in space. In our problem that fixed distance must be 4.5 times the longer wavelength and it must also be 5.5 times the shorter wavelength; $4.5\cdot\lambda_{long} = 5.5\cdot\lambda_{short}$ . This is all we need to know.

P75 In our usual way, write down the condition for interference minima in the reflected light. There is one phase shift ( for the first reflection ).

$$N_B - N_A = m + \frac{1}{2}$$

$$\left[\frac{2t_m}{\lambda/n} + 0\right] - \left[\frac{0}{\lambda} + \frac{1}{2}\right] = m + \frac{1}{2} \tag{P75-1}$$

The thickness and the index of refraction are taken to be the same for each of the wavelengths. This means that there must be a different value of m for each. However, because there are no cases of destructive interference between these two wavelengths, the values for m in the two cases must differ by only 1. Solve (P75-1) for $t_m$.

$$t_m = \frac{(m+1)\lambda}{2n} \tag{P75-2}$$

In (P75-2) the thickness of the film is actually a fixed number – after all, there is only one film. So we can set $t_m = t_{m+1}$. What this means is that the wavelength differs. For a larger value of m we will have a smaller wavelength because the product has to stay unchanged. Using (P75-2) write this down for our two wavelengths.

$$\frac{[m+1](640nm)}{2\cdot 1.58} = \frac{[(m+1)+1](512nm)}{2\cdot 1.58} \tag{P75-3}$$

If I solve (P75-3) for m I can then use (P75-2) to find the thickness of the film.

$$[m+1]\cdot 1.25 = [m+2]$$

$$0.25m = 0.75$$

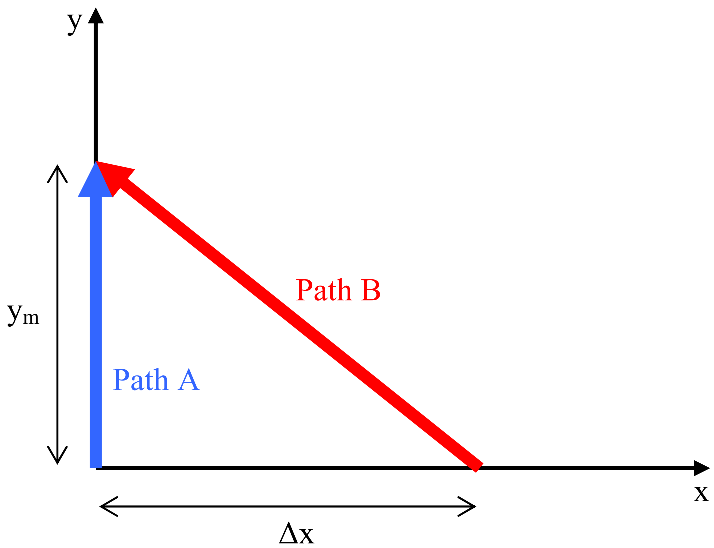
$$m = 3$$

I need to recall that the wavelength that corresponds to m is 640nm; 512mn corresponds to m+1.

$$t = \frac{(3+1)640nm}{2\cdot 1.58}$$

$$= 810.1nm$$

P86 This is a small variation on the two slit problem and we can address it in our usual way. Start by drawing a picture and indicating the two paths. In this case we can start the paths at the antennae draw straight lines to the y axis to join them.




The index of refraction is 1 for both paths.

$$N_A = \frac{y_m}{\lambda}$$

$$N_B = \frac{\sqrt{(\Delta x)^2 + (y_m)^2}}{\lambda}$$

$$N_B - N_A = m + \frac{1}{2}$$

$$m + \frac{1}{2} = \frac{\sqrt{(\Delta x)^2 + (y_m)^2}}{\lambda} - \frac{y_m}{\lambda}$$

We were not given the wavelength but we do know that $c = \lambda \cdot f$. With the frequency and the speed of light in hand we can calculate the wavelength. Combine this with what we have above and solve for $y_m$.

$$\left[\left(m + \frac{1}{2}\right)\left(\frac{c}{f}\right) + y_m\right]^2 = (\Delta x)^2 + (y_m)^2$$

$$\left(m+\frac{1}{2}\right)^2\left(\frac{c}{f}\right)^2+2\cdot\left(m+\frac{1}{2}\right)\left(\frac{c}{f}\right)\cdot y_m+\left(y_m\right)^2=\left(\Delta x\right)^2+\left(y_m\right)^2$$

$$\left(m+\frac{1}{2}\right)^2\cdot c^2+2\cdot\left(m+\frac{1}{2}\right)\cdot c\cdot f\cdot y_m=\left(\Delta x\right)^2\cdot f^2$$

$$y_m=\frac{\left(\Delta x\right)^2\cdot f^2-\left(m+\frac{1}{2}\right)^2\cdot c^2}{2\cdot\left(m+\frac{1}{2}\right)\cdot c\cdot f}$$

$$y_m=\frac{\left(175m\right)^2\cdot\left(6.0\cdot10^6 s^{-1}\right)^2-\left(m+\frac{1}{2}\right)^2\cdot\left(3.0\cdot10^8 m/s\right)^2}{2\cdot\left(m+\frac{1}{2}\right)\cdot\left(3.0\cdot10^8 m/s\right)\cdot\left(6.0\cdot10^6 s^{-1}\right)}$$

$$y_m=\frac{1.1025\cdot10^{18}-9\cdot10^{16}\left(m+\frac{1}{2}\right)^2}{3.6\cdot10^{15}\left(m+\frac{1}{2}\right)}meters$$

$$y_m=\frac{306.25-25\left(m+\frac{1}{2}\right)^2}{\left(m+\frac{1}{2}\right)}meters$$

Because path B is longer than path A, we can only accept non-negative values for m. Furthermore there will be a largest value for m and that will be for the point of destructive interference that is closest to the origin. This is because that is where the difference between the two paths is the greatest. Start with m = 0 and see what we get.

| m | $y_m$ ( meters ) |
|---|---|
| 0 | 600 |
| 1 | 166.7 |
| 2 | 60 |
| 3 | 0 |
| 4 | -44.4 |

It might be helpful to note that the wavelength for 6.0MHz radio waves is 50m. Thus

- the origin is 3.5 wavelengths from the source ( $S_2$ ) at 175m on the x axis and 0 wavelengths from the source ( $S_1$ ) at the origin. ( 3.5 wavelengths difference )

- a point at 60m on the y axis is 3.7 wavelengths from $S_2$ and 1.2 wavelengths from $S_1$. ( 2.5 wavelengths difference )
- a point at 166.7m on the y axis is 4.83 wavelengths from $S_2$ and 3.33 wavelengths from $S_1$. ( 1.5 wavelengths difference )
- a point at 600m on the y axis is 12.5 wavelengths from $S_2$ and 12 wavelengths from $S_1$. ( 0.5 wavelengths difference )
- You could also find corresponding points of destructive interference on the negative y axis ( -60m, -166.7m, -600m). The larger values of m do not correspond to solutions of the problem. If one tries out -44.44m as a solution, the path from $S_2$ is 3.61 wavelengths long. The path from $S_1$ is 0.89 wavelengths long. ( 2.72 wavelengths difference )