# Physics 122
# Chapter 23
# Problem Solutions

Q3 The mirror does not reverse left and right. *We* reverse right and left when we turn around to face the other direction. Compare this to up and down. We do not flip upside down when we face the other direction and so we see the mirror as doing the right thing when it reflects our head to our head and our feet to our feet. In the horizontal plane the mirror merely reflects right hand to right hand and left to left.

Actually there is something that the mirror does reverse. Suppose that you are facing north. If you stick out your right hand it will be pointing to the east. Now let's tape some little markers to our body. A little Nasty brown marker ( for north ) is taped to your nose. A stick with a Silver ( for south ) marker is taped to the back of your head so that pokes up above your hair. The top of your head gets a Teal marker and your feet get a Black marker. Your right hand has an Ecru marker and your left gets one that is Wistfully yellow. When you look at your reflection in the mirror, top, bottom, east, and west are all pointing in the correct directions. North and south are switched though. That is what a mirror really does – it inverts front to back while leaving the rest along. When a person turns around to face you they switch front to back with a rotation that also switches left and right.

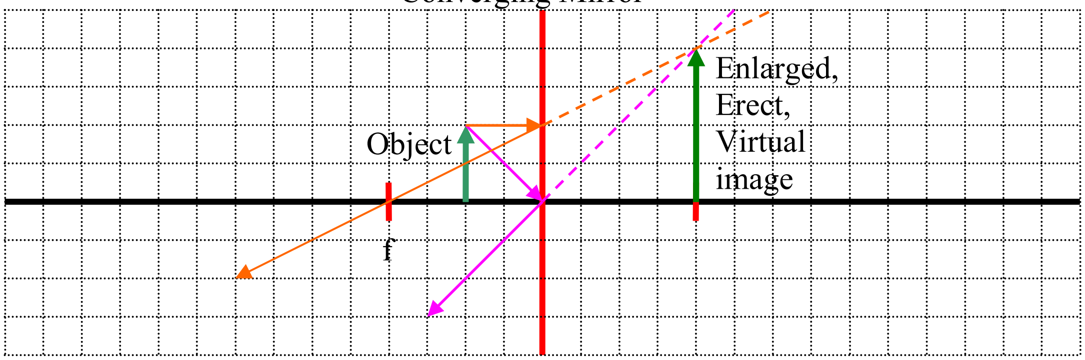
Q8 In any small patch of surface the water has parts that tilt in many angles near to the horizontal. If we are looking at a moon that is low in the sky, it is likely that some of these parts will have the correct tilt to reflect light to us. Thus we see light from long streak of water. As the waves typically move toward the shore, these tilted patches do not send light to the side and so the lighted patch is long and narrow.

Q10 Simply send light in at, say, 45° to the surface and then use Snell's law. The index of refraction for air is pretty nearly one. So if the refracted angle is θ then the speed of light in the material is

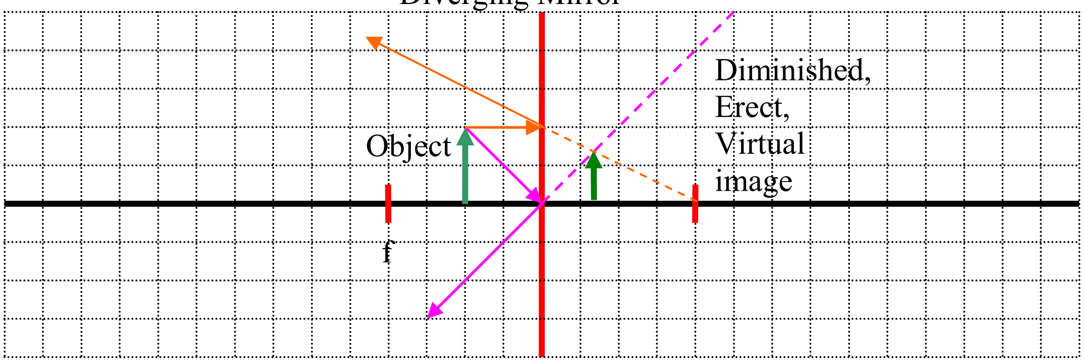
$$c_{material} = c_{vacuum} \frac{\sin(\theta)}{\sin(45^\circ)}$$

Q19 See figure 23-46 on page 658. The image of the face is erect and enlarged. Diverging mirrors only produce images that are smaller than the objects and so this must be the case of an object ( the face ) between the converging mirror and its focal point. Examine the following ray diagrams to see this.

Converging Mirror

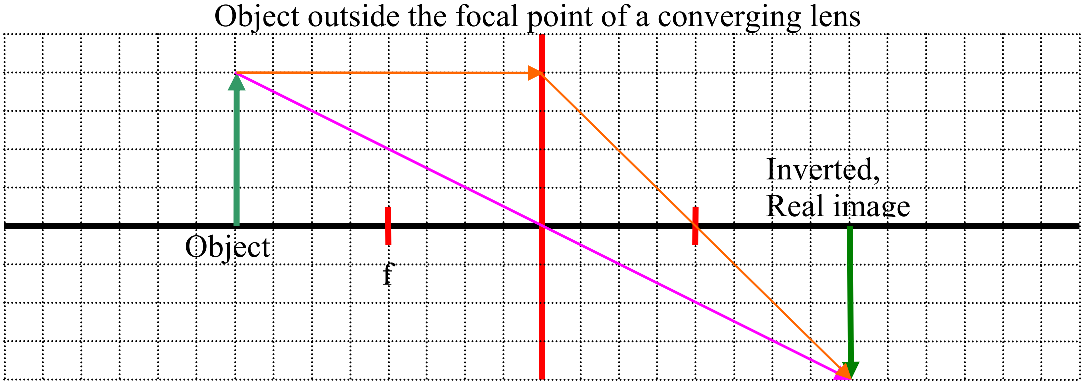



Diverging Mirror



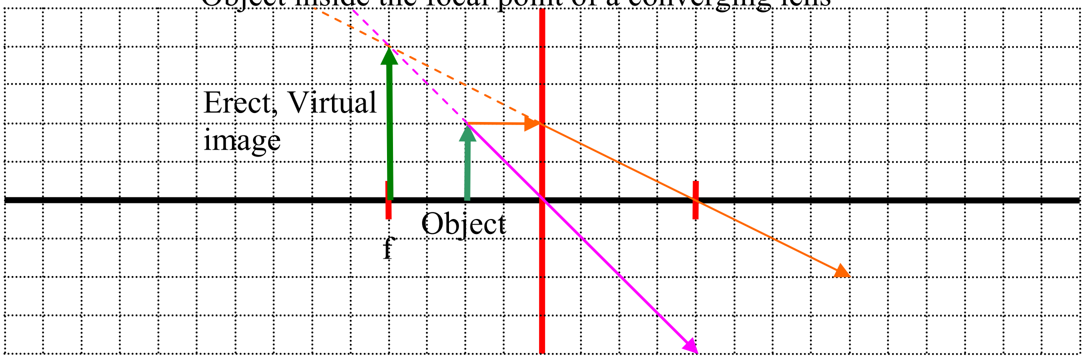

Q22 There are three cases: inside and outside the focal point of a converging lens and anywhere in front of a diverging lens.

Object outside the focal point of a converging lens



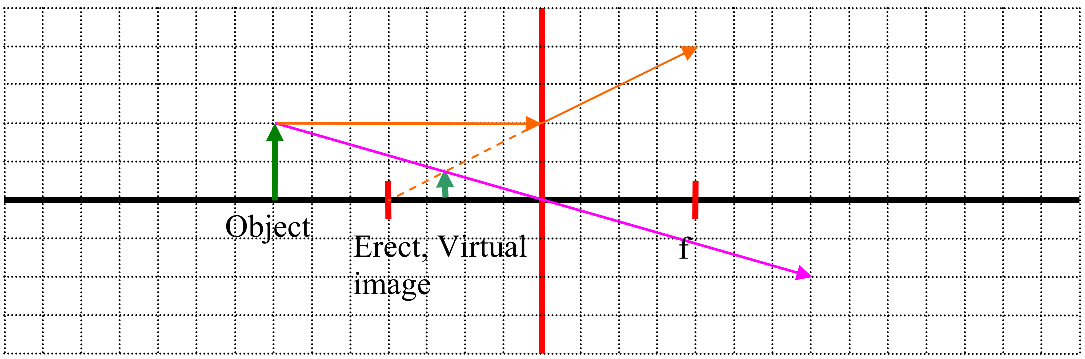

Converging lens – object outside the focal point. The rays drawn must cross to the right of the lens ( real image ) but cannot meet above the optical axis ( inverted image ).

Object inside the focal point of a converging lens




Converging lens – object inside focal point. The rays cannot cross to the right of the lens ( virtual image ). Their projections can only cross above the axis ( erect ).

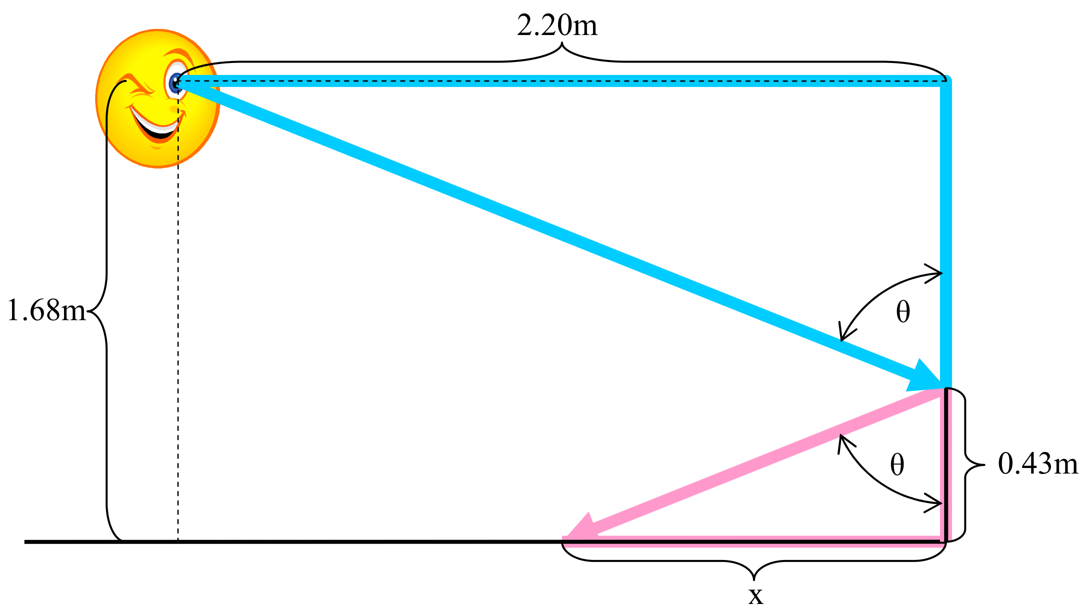
Object to the left of a diverging lens




Diverging lens. The rays cannot meet to the right of the lens. The projections of the rays must meet between the object and the lens. As the pink ray is necessarily above the optical axis in this region, the image will be erect. Of course, it is also virtual as, aside from the pink ray, none of the light is actually passing through this point.

P4 I have reproduced the figure from page 658. Apply the law of reflection to the light that reflects from the mirror so that two similar triangles are formed[*].

[*] The two angles marked $\theta$ are the same by the law of reflection and both triangles are right triangles. Thus they are similar triangles.




Because these triangles are similar, the ratio of their altitudes is the same as the ratio of their bases.

$$\frac{x}{2.20m} = \frac{0.43m}{(1.68m - 0.43m)}$$

This is easily solved; x = 0.76m.

P9 If the ornament has a radius of 4.5cm then its focal length will be one half of that and negative[†]; f = -2.25cm. The object distance is given as 30cm. Bring out the thin lens formula.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

As we are often given the object distance and the focal length of the lens there is a handy bit of algebra we can do and then save some work in the future. Solve the thin lens equation for the image distance.

$$d_i = \frac{f \cdot d_o}{d_o - f} \qquad \text{(P9-1)}$$

Now use (P9-1) to find the image location relative to the surface of the ornament[‡].

[†] See the second paragraph on page 636 and equation 23-1 in your text. See also the first paragraph on page 641.

$$d_i = \frac{(-2.25cm)(30cm)}{30cm-(-2.25cm)}$$
$$= -2.09cm$$

As we saw in class, an object placed anywhere on the front side of the mirror must produce a virtual and erect image.

P14 This is rather similar to P9. We are given that $2 \cdot h_i = h_o$ and that $d_o = 3m$. This should let us get the image distance if we need it.

$$\frac{d_i}{d_o} = -\frac{h_i}{h_o}$$
$$d_i = -(3m)\left(\frac{1}{2}\right) \qquad \text{(P14-1)}$$
$$= -1.5m$$

How can we tie this to the radius of curvature? If you haven't read it, look at first footnote for P9. The radius of curvature is twice the focal length of the spherical mirror and so to find the focal length of the mirror is to find the radius of curvature. Use the thin lens equation. I will solve it for the focal length.

$$f = \frac{d_o \cdot d_i}{d_o + d_i} \qquad \text{(P14-2)}$$
$$f = \frac{(3m)(-1.5m)}{(3m)+(-1.5m)}$$
$$= -3m$$

Thus the radius of curvature is -6m.

P25 The index of refraction is the ratio of the speed of light in vacuum to that in the material in question. The index of refraction for water is 4/3 so we have in this case,

‡ Because we are not looking at just a tiny portion of the surface, the thin lens approximation is not so great anymore. A part of that is a clear inability to say, in the horizontal direction, just where the surface is.

$$n_{material} = \frac{c_{vac}}{c_{material}}$$

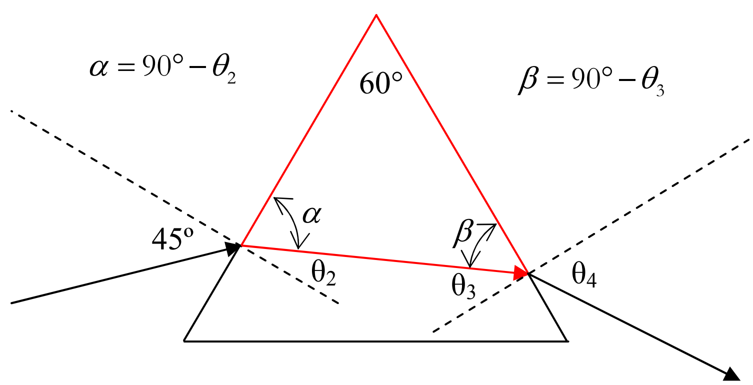
$$= \frac{c_{vac}}{0.89 \cdot c_{water}}$$

$$= \frac{c_{vac}}{0.89 \cdot \left( c_{vac} / 4/3 \right)}$$

$$= \frac{4}{3 \cdot 0.89}$$

$$= 1.49$$

P23 The solution to this problem rests on the application of Snell's law and a little geometry. Recall that the index of refraction for air is 1.



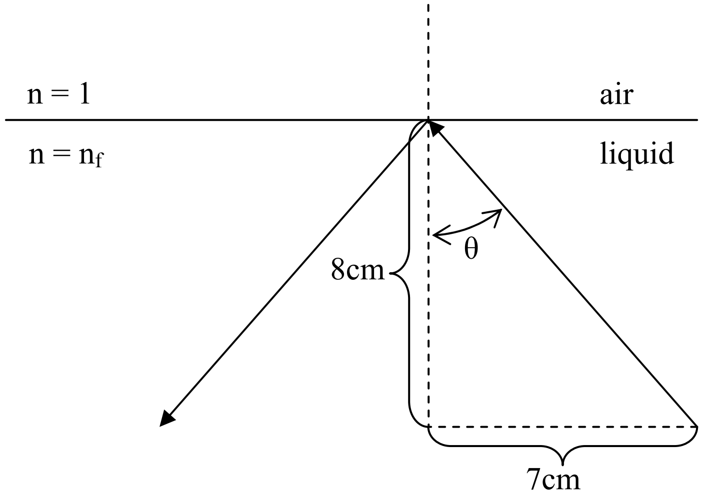

$$\sin(45°) = n\sin(\theta_2)$$
$$n\sin(\theta_3) = \sin(\theta_4)$$

$$\alpha + \beta + 60° = 180°$$
$$\left(90° - \theta_2\right) + \left(90° - \theta_3\right) + 60° = 180°$$

The first two lines are Snell's law applied to the entrance and exit of the light from the prism. The last two sum the angles of the red triangle and require that they add to 180°. One can solve the first equation for $\theta_2$ and then with that in hand solve the third equation for $\theta_3$. The second equation will then yield

$$\theta_4 = 60.5°.$$

P39 Let's start with a picture of the problem. I am assuming that there is air above the liquid; were this not the case we would not know how to proceed as the index of refraction for the upper medium is helps determine when total internal reflection occurs.




Because total internal reflection is occurring we know that $\theta \geq \theta_{critical}$. For angles between 0° and 90° the sine of a larger angle is bigger than the sine of a smaller angle; thus

$$\sin(\theta) \geq \sin(\theta_{critical}) \tag{P39-1}$$

Let's put into (P39-1) what we know about each of the sine functions.

$$\begin{aligned} \sin(\theta_{critical}) &= \frac{n_{air}}{n_f} \\ &= \frac{1}{n_f} \end{aligned} \tag{P39-2}$$

The hypotenuse of the triangle is $\sqrt{(7cm)^2 + (8cm)^2} = 10.63cm$ so we can write

$$\begin{aligned} \sin(\theta) &= \frac{7cm}{10.63cm} \\ &= 0.659 \end{aligned} \tag{P39-3}$$

Inserting from (P39-3) and (P39-2) into (P39-1) we have

$$0.659 \geq \frac{1}{n_f}$$

or

$$n_f \geq 1.52$$

P53 The easiest way to work through this problem is to note that we have information about focal length and magnification and we are asked to find object distances. There are two relevant expressions. These are the lens equation

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i} \qquad \text{(P53-1)}$$

and the relationship of magnification to the image and object distances[§].

$$m=-\frac{d_i}{d_o} \qquad \text{(P53-2)}$$

I do not know anything about image distances here and so I will solve (P53-1) with (P53-2) for $d_o$ while eliminating $d_i$.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{-m\cdot d_o}$$

$$\frac{1}{f}=\frac{1}{d_o}\left(1-\frac{1}{m}\right) \qquad \text{(P53-3)}$$

$$d_o=f\cdot\left(1-\frac{1}{m}\right)$$

So for part a) with a 5cm focal length and a magnification[**] of -2 the object distance is 7.5cm. For part b), when we change the magnification[††] to +2 we calculate an object distance of 2.5cm.

P56 This problem is rather like P53. But now the thing I want is the distance between the image and the object. Recall that the sign convention for these distances measures positive distance for the object to the left and positive distance for the image to the right ( or visa-versa if you prefer ). Thus the distance between them is given by $d_o+d_i$ and not $d_i-d_o$.

I will create the sum of the distances by using the lens equation and the magnification. A useful result is (P53-3).

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[§] See page 638 equation 23-3

[**] The negative magnification tells us that the image distance is positive and so the image is real. This only follows is the object distance is positive.

[††] The positive magnification tells us that the image distance is negative and so the image is virtual. This only follows is the object distance is positive.

$$d_o = f \cdot \left(1 - \frac{1}{m}\right) \tag{P56-1}$$

Then I do the same calculation except that I eliminate the object distance instead of the image distance. Work through the details!

$$d_i = f\left(1 - m\right) \tag{P56-2}$$

Then I add (P56-1) to (P56-2).

$$d_i + d_o = f\left(2 - m - \frac{1}{m}\right)$$

For this problem the focal length is 75cm and the magnification is -2.5. This gives a separation of 368cm.

P59 Let's work this one through without the ray diagram. The original object at infinity produces an image at the focal point of the first lens. That is 20cm behind the converging lens and 6cm behind the diverging lens. Thus we have a virtual object at a location of -6cm for the diverging lens. Now use the lens equation (P9-1) with this object distance and the focal length of the diverging lens.

$$d_i = \frac{f \cdot d_o}{d_o - f}$$

This location will be relative to the diverging lens. $d_i = 7.4cm$

P64 This is similar to P59 but now we must do a bit of work to find out where the image from the first lens is formed. Use the lens equation (P9-1) to locate the image from the converging lens.

$$\begin{aligned} d_i &= \frac{f \cdot d_o}{d_o - f} \\ &= \frac{(60cm)(20cm)}{60cm - 20cm} \\ &= 30cm \end{aligned} \tag{P64-1}$$

As the lenses are 25cm apart, this is 5cm to the right of the second lens. It becomes a virtual object with a location of -5cm for the second lens. Use the lens equation again.

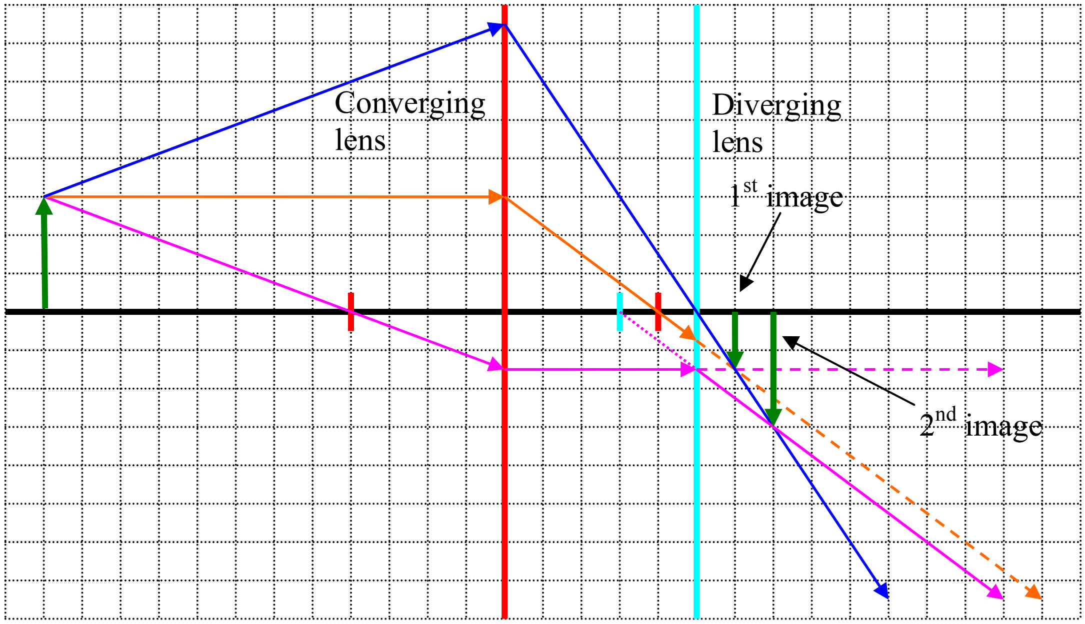
$$
\begin{aligned}
d_i &= \frac{f \cdot d_o}{d_o - f} \\
&= \frac{(-5cm)(-10cm)}{(-5cm)-(-10cm)} \qquad \text{(P64-2)} \\
&= 10cm
\end{aligned}
$$

The final image is 10cm to the right of the second lens. The magnification is found by multiplying the magnifications for the individual lenses.

$$
\begin{aligned}
m_1 &= -\frac{30cm}{60cm} \\
&= -0.5 \\
m_2 &= -\frac{10cm}{-5cm} \\
&= 2 \\
m_T &= m_1 m_2 \\
&= -1
\end{aligned}
$$

The final image is the same size as the original object. It is inverted and, because the last image distance is positive, it is a real image. Can you ever form a real image to the left of the final lens[‡‡]? Can you ever form a virtual image to the right of the final lens[§§]? Convince yourself of the answers.

[‡‡] No
[§§] No




The orange and the pink rays are used to find the first image. Once that is done, the dark blue ray can be drawn. It is chosen so that it will pass through the center of the 2nd lens***. The pink ray is parallel to the optical axis when it encounters the diverging lens and so we can draw the refracted pink ray. The dashed pink line only shows where the pink ray would go in the absence of the second lens. We used that to locate the first image.

The solid pink line and the dark blue line are each rays as the second lens sends them along. Where they meet is the location of the final image.

In this diagram each square is 5cm on a side. You can see that the image is 10cm to the right of the diverging lens, that it is the same size as the original object and inverted, and that it is real.

P89 Sometimes I assign problems that are longer and more algebraically intense. This is one such. My goal to push your developing abilities somewhat and, I hope, to give you a sense of satisfaction when you complete the task. If you made it through this problem then you have come a fair ways in you analytical abilities since you started the class a semester and a half ago.

Be sure to think about the way in which the solution develops as you read through this. The reasons for deciding to try each thing are far more important than the solution itself.

*** Place your ruler so that the tip of the first image and the center of the second lens touch the edge. Draw the line back to the first lens. At that point bend the ray so that it comes from the original object.

This seems like a problem we should be able to do without drawing a ray diagram. We know the following: Here I am trying to make sense of what I have been told. What do the words mean and what is their significance. I make this list so that, having thought it through once, I do not have to think it through a second time.

- The focal length is positive. This is true because we have a converging lens.
- The focal length is less than 0.6m. We know this because an object placed inside the focal point of a converging lens produces a virtual image not a real image. Indeed the focal point is less than 40cm because when the object is moved closer to the lens the image is still real and shifts farther away from the lens.
- The difference between the image distances when the object is at 40cm and when it is at 60cm is 10cm.

Now try to find a connection to the requested answer.

On the basis of this information we are to find the focal length of the lens. All of this information concerns focal length, image distance, and object distance. There is only one thing that relates these quantities and that is the thin lens equation. As we don't have image distances directly, I think that I will want to be able to make those go away and so I will try writing down (P64-1) for each placement of the object.

$$d_i = \frac{f \cdot d_o}{d_o - f} \tag{P89-1}$$

For the object at 60cm (P89-1) becomes

$$d_{i,60} = \frac{f \cdot 60cm}{60cm - f}. \tag{P89-2}$$

For the object at 40cm (P89-2) becomes

$$d_{i,40} = \frac{f \cdot 40cm}{40cm - f} \tag{P89-3}$$

And from the third bulleted point above I have that $d_{i,40} - d_{i,60} = 10cm$. This is such a useful skill! So very many times you are to find quantity or you know about some quantity but you do not have an expression that gives that thing. So you just build it! Here "the thing" was the difference in the two image distances. Fill into this from (P89-2) and (P89-3).

$$10cm = \frac{40cm \cdot f}{40cm - f} - \frac{60cm \cdot f}{60cm - f} \tag{P89-4}$$

Now I need to solve this for *f*. Don't get excited. Just get some paper and proceed slowly and carefully. There is nothing that needs doing here that you haven't done many times before.

$$10cm = \frac{(40cm \cdot f)(60cm - f)}{(40cm - f)(60cm - f)} - \frac{(60cm \cdot f)(40cm - f)}{(60cm - f)(40cm - f)}$$

$$10cm = \frac{\left[2400cm^2 \cdot f - 40cm \cdot f^2\right] - \left[2400cm^2 \cdot f - 60cm \cdot f^2\right]}{2400cm^2 - 100cm \cdot f + f^2}$$

$$10cm\left(2400cm^2 - 100cm \cdot f + f^2\right) = 20cm \cdot f^2$$

$$2400cm^2 - 100cm \cdot f + f^2 = 2 \cdot f^2$$

$$0 = f^2 + 100cm \cdot f - 2400cm^2$$

Use the quadratic formula.

$$f = \frac{-100cm \pm \sqrt{(-100cm)^2 - 4 \cdot 1 \cdot \left(-2400cm^2\right)}}{2}$$

$$f = \frac{-100cm \pm 140cm}{2}$$

$$f = \begin{cases} 20cm \\ or \\ -120cm \end{cases}$$

The 20cm focal length is consistent with the first two bulleted points but the -120cm focal length is not. Therefore I say that the focal length must be 20cm.