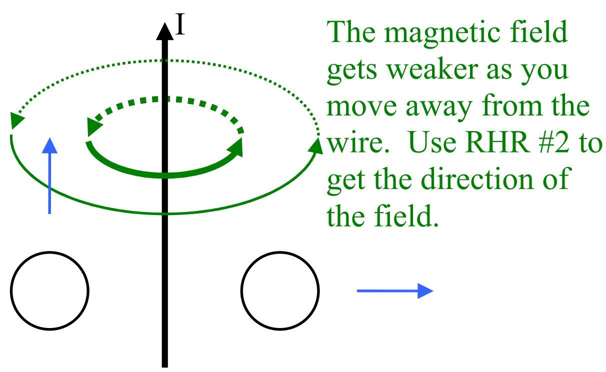
Physics 122
Chapter 21
Problem Solutions

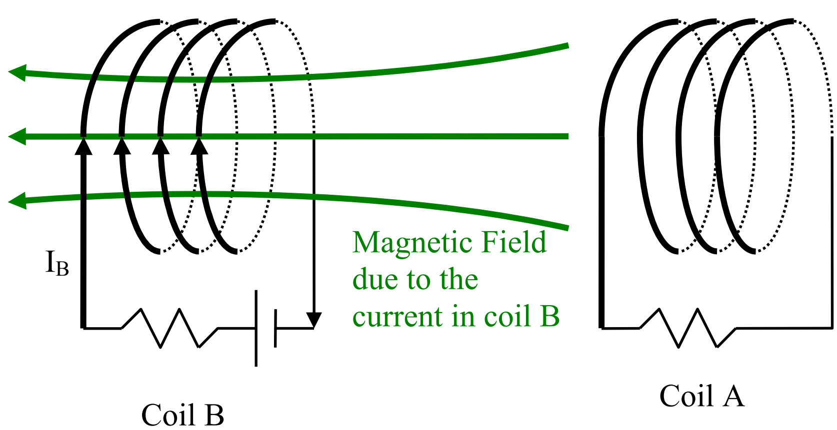
Q4 There will be a current if an EMF exists to drive it. The EMF is caused by a changing magnetic flux through the loop. Thus we need to be able to describe the magnetic field through which the loop is moving.



The left hand loop moves parallel to the wire. Through any given part inside the loop the magnetic field remains constant in magnitude and direction. The flux is therefore constant and the EMF is zero. There is no current in the loop.

The right hand loop is in a magnetic field that is directed into the paper through the area enclosed by the loop. The magnitude of the field diminishes as the loop is pulled away from the current. Thus, to cancel the change, the induced field will also be into the paper through the middle of the loop. Right Hand Rule #2 indicates that an induced current that is clockwise will produce a field directed this way.

Q6 To understand this example I need to figure out what the drawing means. I will reproduce it here with some dashed lines to represent the back side of the coil.



In my picture the magnetic field in coil A due to the current in coil B will generally be in the right to left direction. Use RHR #2 on a front part of one of the loops to establish this.

A) If coil B moves closer to coil A then the strength of the magnetic field inside coil A will increase. To offset this change, there will need to be an induced field in coil A that points from left to right. To produce this, a current in coil A will need to be in the downward direction through the front of the loops in the coil. In the resistor, this will move from left to right.
B) If coil B moves away from coil A then the magnetic field will still point from right to left. However, the field is now growing weaker and to compensate for this change the induced field will also point from right to left. To produce a field in this direction through coil A, the current in coil A must be in the upward direction through the front of the loops. This requires a current in the resistor that moves from right to left.
C) If the resistance in coil B is increased then the current in coil B will decrease and the strength of the magnetic field will also decrease. The effect on coil A will be the same as in B).

Q12 What happens when a conductor moves through a magnetic field? The Lorentz force law does not tell us anything because the conductor is neutral. While the positive charges would be pushed in one direction and the negative charges pushed in the other, they are all tied together in the bit of metal and so the net force due to the simple movement of the metal in the field is zero.

We did see in class that the movement of a conductor into or out of a magnetic field produces a current in the metal and that this current results in a force so long as only a part of the metal is in the field. The force resisted the motion of the metal both upon entering and upon exiting the field. The metal would slow down relative to a similar bit of non-metal. If everything slid down a low friction incline and then off the edge of a table, and the metal bits were traveling more slowly at the bottom edge of the incline then the two types of materials would fall into two piles on the floor. It seems that you would be best served by a series of magnets so that there would be many boundaries to cross.

Q20 Inductance is all about sharing a magnetic field. For the largest mutual inductance you want all of the field that passes through one loop to pass through the other loop. Just place one next to the other so that they share a common axis*.

For a small mutual inductance, rotate the axis of one loop so that the two axies are perpendicular. If you can arrange to have the centers of the two coils coincide as well then the magnetic field generated by one of the loops will not pass through the other at all.

* Like two plates stacked up.

Q21 The time constant for an LR circuit gives the time for the current to reach about 63% of its maximum value. There is no reference to the magnitude of that maximum current. However, the maximum value is simply the battery voltage divided by the resistance so that a given fraction of the maximum will correspond to different currents if you use different batteries. If you want to get to 6.3A and the maximum current is only 5A you will be waiting a long time. If, on the other hand, the maximum current is 10A you will get there in a time equal to L/R.

P1 This problem is about understanding the form of Faraday's law.

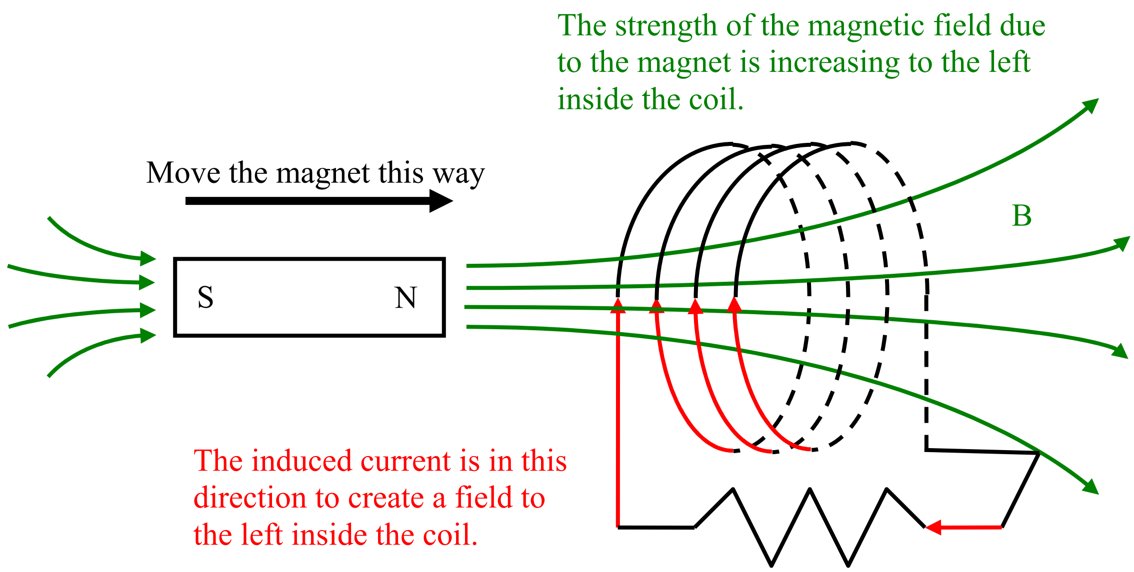
$$EMF = -\frac{\Delta \phi_B}{\Delta t}$$

The change in flux is +38Wb – (-50Wb) = 88Wb. The change in time is 0.42s.

$$EMF = -\frac{88Wb}{0.42s}$$
$$= -209.5V$$

The negative sign only has meaning in the geometrical context of the path that defines the loop. If you travel the path in one direction the flux values are as given and the electric field is, on the whole, in the opposite direction you travel. If you had chosen to move around the path in the opposite direction, the electric and magnetic fields would be unchanged but the flux values would be the negative of those given. This is why I have you work with the magnitude of the EMF and use Lenz's law to establish the direction of the current.

P3 By convention we label the end of the magnet from which the field emerges as the North pole. Thus, the picture can be drawn this way.



The flux is increasing as the magnet approaches and the induced magnetic field tends to offset this change. This requires the induced field to be to the left through the middle of the coil. A current moving from right to left through the resistor will accomplish this. Use RHR #2 to see this.

P6 This problem is just like P1 except that we now have to convert area and magnetic field into flux. We are free to choose up or down as the positive direction for the magnetic field but once the choice is made we have to stick to it. I will choose the initial direction of the magnetic field (up) to be positive for both field and area. Thus the initial flux is

$$\begin{aligned}\phi_B &= \vec{B}\bullet\vec{A}\\ &= (0.63T)\left(\pi\left(\frac{0.102m}{2}\right)^2\right)\cos(0°)\\ &= 497mWb\end{aligned}$$

and the final flux is

$$\begin{aligned}\phi_B &= \vec{B}\bullet\vec{A}\\ &= (0.25T)\left(\pi\left(\frac{0.102m}{2}\right)^2\right)\cos(180°)\\ &= -197mWb\end{aligned}$$

Now proceed as in P1 to get the magnitude of the EMF equal to 4.63V.

P8 We are interested only in the magnetic flux through the small loop; changes in this flux are what lead to currents in the small loop. Initially, the current in the larger loop has some value and it circulates in the counter-clockwise direction. Using RHR #2 we can see that the magnetic field inside the large loop, and therefore inside the small loop as well, is out of the page.

When the resistance in the large loop is increased it reduces the current in the large loop and therefore reduces the size of the magnetic field. The small loop sees a decreasing flux and Lenz's law indicates that a current will be induced to offset the change. To accomplish this I need an induced field that is out of the page through the inside of the small loop. A counter-clockwise induced current will do this.

Had this entire process happened with the small loop located outside the large loop the only change would have been the direction of the magnetic field due to the large loop. In the plane of the paper this field is directed into the paper everywhere outside of the large loop. If this field were diminishing in size due to the increasing resistance in the large loop the effect on the small loop would be unchanged except for the direction of the current. The induced field would now need to point into the paper and so the induced current would be clockwise.

P12 This problem is all about the material in the first two paragraphs of section 21-3. For me, it is easier to think about the moving rod in the absence of the U shaped conductor because then I do not have to worry about what the conductor is contributing to the analysis. If I take this approach then I have to abandon Faraday's law because I do not know how to apportion the EMF to the different parts of the loop.

The rod moves to the right at a speed v and this means that the electrons are moving with that velocity too. Once the electrons reach steady state – that is they settle down and stop moving – then the net force on them must be zero. What forces are present? Turn to the Lorentz force law. $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$. With the velocity to the right and the magnetic field out of the paper $\vec{v} \times \vec{B}$ is toward the bottom of the page. (Did I get the direction right? Check it and see.) For the net force to be zero the electric field must be toward the top of the page and its size must be $\left|\vec{v} \times \vec{B}\right|$. I can now answer the second question.

$$\begin{aligned} \vec{E} &= -\vec{v} \times \vec{B} \\ \left|\vec{E}\right| &= \left(0.15 m/s\right)\left(0.8T\right) \\ &= 0.12 V/m \end{aligned}$$

There is no reference to position along the length of the bar in the previous argument so the electric field must be constant along the entire length of the bar. That makes it easy to calculate the EMF[†] between the ends of the bar. If I start at the bottom of the bar and do my sum of $\vec{E} \bullet \Delta\vec{l}$ along the entire bar I just get E multiplied by the length of the bar. The electric field points toward low potential so the bottom end of the bar is at a potential $\left(0.12 V/m\right)\left(0.12 m\right) = 14.4 mV$ higher than the top end of the bar.

Suppose that you really wanted to use Faraday's law to work this problem; how should you think about it? You have to accomplish two things.

1. You need a well defined loop so that you can do the EMF sum.
2. You need to arrange the loop so that all of the EMF is spread over just the bar and it is done so in a uniform manner.

The difference here is that you will have a current flowing in this setup whereas in the prior analysis there was only a static charge at each end of the rod.

[†] I really don't like the fact that the author uses EMF here. This really is a change in potential ΔV. The EMF is defined for an entire loop and we don't have that here.

The drawing in Figure 21-12 part a) actually is just what you need. You assert that the U shaped conductor is a perfect conductor. No electric field is needed to keep the current moving through it. Then you make the bar have some uniform resistance along its length. Now when you do the sum of $\vec{E}\bullet\Delta\vec{l}$ along the U there is no E to contribute. All of the EMF must come from an electric field in the bar. At this point you can proceed as in the text and get the same result. As I wrote above, I am not fond of this analysis – it seems contrived.

P18 If I solve part a) of this problem I should be able to use Joule's law to do part b). There will be a current in the coil because there will be an EMF to drive it. The EMF is due to the changing magnetic flux through the coil. The connection between the EMF and the resulting current is Ohm's law so I will need the resistance of the coil as well.

- Faraday says I need $\frac{\Delta\phi_B}{\Delta t}$.
  - $\frac{\Delta\phi_B}{\Delta t}=\frac{\Delta\vec{B}}{\Delta t}\bullet\vec{A}+\vec{B}\bullet\frac{\Delta\vec{A}}{\Delta t}$
    - $\frac{\Delta\vec{A}}{\Delta t}=0$
    - $\left|\frac{\Delta\vec{B}}{\Delta t}\right|=8.65\cdot10^{-3}T/s$ The direction is unimportant as we need only the size of the current and not its direction.
    - $\left|\vec{A}\right|=N\pi r^2$ The N refers to the number of turns in the coil. The electric field pushes charge in every loop of the coil and so the total EMF gets bigger as you have more loops. $\left|\vec{A}\right|=0.76m^2$
  - $\left|EMF\right|=\left|\frac{\Delta\phi_B}{\Delta t}\right|=6.58mV$
- $I=EMF/\mathrm{R}$
  - $\mathrm{R}=\rho\frac{L}{A}$ ‡
    - Here ρ is the resistivity of copper ($1.68\cdot10^{-8}\Omega\cdot\mathrm{m}$),
    - L it the total length of the wire in the loop (13.8m), and
    - A is the cross-sectional area of the wire itself ($5.31\cdot10^{-6}\mathrm{m}^2$)
  - $\mathrm{R}=4.37\cdot10^{-2}\Omega$
- $I=0.15A$

  Now I use Joule's law to find the rate at which the coil is heated. $P=I\cdot EMF$
- $P=9.92\cdot10^{-4}W$

‡ See section 18-4

P33 The ratio of the voltages is the same as the ratio of the number of turns in each coil. In the primary coil the voltage is 240V and in the secondary it is 12,000V. The ratio is 50. The secondary coil should have 50 times the number of turns that the primary does. If you put the transformer in backwards then it would reduce the voltage by a factor of 50. The output coil would have a voltage of 4.8V.

P35 The one thing you need in this problem is Joule's law. We are told that the power put into or out from this transformer is 95W. If the input current is 22A then

$$P = I_{in}V_{in}$$
$$V_{in} = \frac{95W}{22A}$$
$$V_{in} = 4.32V$$

Because the output voltage is 12V this is a step up transformer. It increases the voltage by a factor of 2.78.

P40 Apply our definition of inductance. The algebraic signs do not matter as we want only the size of the inductance and not the direction of an induced EMF.

$$|V_L| = L \cdot \left|\frac{\Delta I}{\Delta t}\right|$$
$$L = \frac{2.5V}{\left(59 \cdot 10^{-3} A \big/ 12 \cdot 10^{-3} s\right)}$$
$$L = 0.51H$$

P44 The point of the resistance in the coil is that you have a voltage drop in the direction of the current given by Ohm's law. The voltage drop due to the changing current will be in the direction of the current if the voltage is increasing (the inductor is trying to prevent the increased current flow) and in the opposite direction to the current if the current is decreasing.

Think of our basic LR circuit when we closed the switch to start it up. There was a voltage rise at the battery and there were voltage drops at both the inductor and the resistor. If, for some reason, the current began decreasing then the inductor would begin to act as a battery by taking energy out of the magnetic field and using it to push the current along.

So, in this case with the current increasing, the voltage drop is the sum of IR and $L\Delta I/\Delta t$.

$$V = (3A)(2.25\Omega) + (440 \cdot 10^{-3} H)(3.5A/s)$$
$$= 8.29V$$

Suppose that we had this same problem but the current was 3A and decreasing at the rate of 3.068A/s. The voltage drop across the inductor would be zero! What does this mean? If you put a voltmeter across the terminals of the inductor there would be no voltage difference. You could also hook up a resistor between the ends of the inductor and no current would flow through it. The collapsing magnetic field would be producing just the amount of power needed to push 3A through the resistance.

P49 The initial part of this problem is just a matter of multiplying an energy density by a volume to get a total energy. We will make the comparisons after that is done.

The energy density is given by the expression $B^2/2\mu_0$. With the supplied field and a value of μ_0 from the inside cover of the textbook we find the energy density to be $9.95\cdot10^{-4}$J/m^3. What is the volume of the spherical shell 10km thick with an inner radius equal to that of the Earth?

There are two ways you might think of this. One is to take volume of the larger sphere and subtract the volume of the smaller sphere leaving the volume of the shell that was between them. This is perfectly correct. A second idea would be to take the surface area of the Earth and multiply that by the 10km thickness of the shell. The second idea is an approximation to be sure. How good of an approximation is it? Let's calculate the exact result and then compare.

Exact volume

$$\begin{aligned} V_{shell} &= V\left(R_{Earth} + 10km\right) - V\left(R_{Earth}\right) \\ &= \frac{4\pi}{3}\cdot\left(6380km + 10km\right)^3 - \frac{4\pi}{3}\cdot\left(6380km\right)^3 \\ &= 5.123\cdot10^9\, km^3 \\ &= 5.123\cdot10^{18}\, m^3 \end{aligned}$$

Approximate volume

$$\begin{aligned} V_{shell} &\simeq \left(\text{Surface area of the Earth}\right)\left(\text{Thickness of shell}\right) \\ &\simeq \left(4\pi\cdot\left(6380km\right)^2\right)\left(10km\right) \\ &\simeq 5.115\cdot10^9\, km^3 \\ &\simeq 5.115\cdot10^{18}\, m^3 \end{aligned}$$

A difference of 8 parts in 5000 is pretty good. The total energy is $5.09\cdot10^{15}$J.

Now let's compare this to the Kinetic energy of the Earth as it spins on its axis and as it revolves around the sun. First I find the rotational KE.

$$
\begin{aligned}
KE &= \frac{I\omega^2}{2} \\
&= \frac{1}{2}\cdot\left(\frac{2}{5}\cdot 5.98\cdot 10^{24}\,kg\cdot\left(6.380\cdot 10^{6}\,m\right)^2\right)\left(\frac{2\pi rad}{day}\cdot\frac{1day}{24hours}\cdot\frac{1hour}{3600s}\right)^2 \\
&= 2.57\cdot 10^{29}\,J
\end{aligned}
$$

Now find the translational KE. The speed of the Earth is the circumference of the orbit divided by its period.

$$
\begin{aligned}
KE_{orbital} &= \frac{mv^2}{2} \\
&= \frac{1}{2}\cdot\left(5.98\cdot 10^{24}\,kg\right)\left(\frac{2\pi\cdot 1.496\cdot 10^{11}\,m}{\left(365.25days\right)\left(\frac{24\cdot 3600s}{1day}\right)}\right)^2 \\
&= 2.65\cdot 10^{33}\,J
\end{aligned}
$$

You may have heard that the Earth's magnetic field collapses and then reverses its direction every few hundred thousand years. Would you expect this to affect the motion of the Earth?

P50 When you have just closed the switch the current is zero. This is rather like asking what your velocity is immediately upon starting from rest. If the current is zero then the voltage drop across the resistor is zero as well. Kirchhoff's loop law then tells us that the voltage drop across the inductor must be all of the battery voltage.

$$|V_L| = L\left(\frac{\Delta I}{\Delta t}\right)$$

So at t = 0

$$
\begin{aligned}
V_B &= L\left(\frac{\Delta I}{\Delta t}\right)_0 \\
\left(\frac{\Delta I}{\Delta t}\right)_0 &= \frac{V_B}{L}
\end{aligned}
$$

As the initial current was zero and we are to pretend that the current continues to increase at this rate, the current as a function of time would be

$$I(t) = \left(\frac{\Delta I}{\Delta t}\right)_0 \cdot t$$
$$= \frac{V_B}{L} \cdot t$$

At what time t' would it reach the maximum current value of V_B/R?

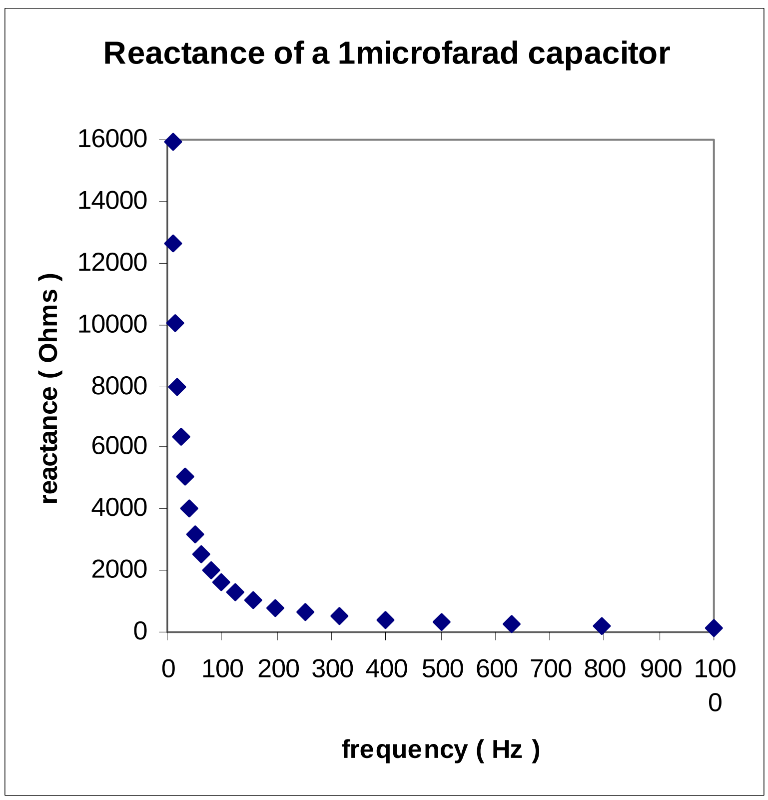
$$\frac{V_B}{R} = \frac{V_B}{L} \cdot t'$$
$$t' = \frac{L}{R}$$

This is just the time constant for an LR circuit.

P56 Recall that ω = 2πf.

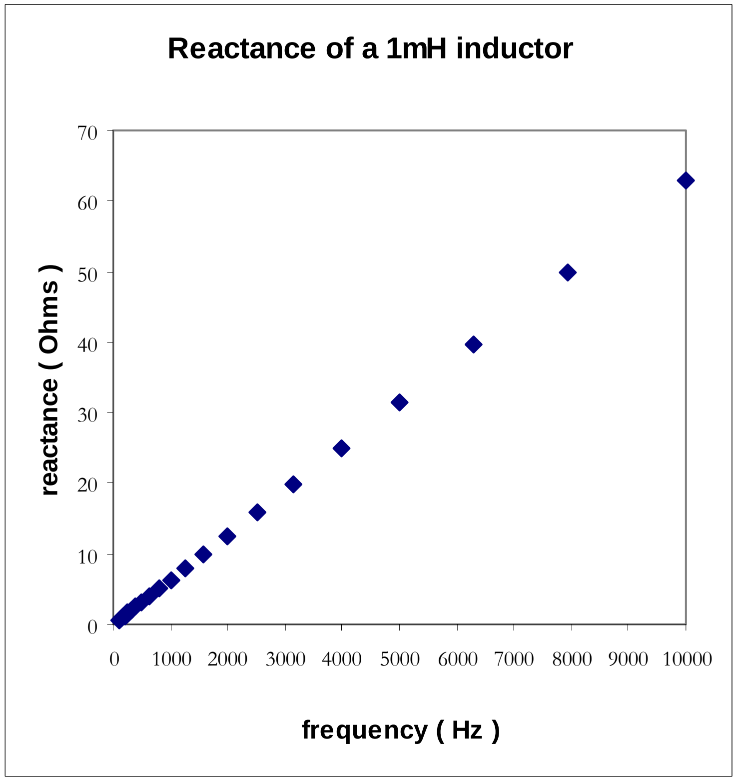
$$|X_C| = \frac{1}{2\pi f \cdot 10^{-6}} \Omega s^{-1}$$

Frequency (Hz)	Reactance (Ohms)
10	15916
13	12642
16	10042
20	7977
25	6336
32	5033
40	3998
50	3176
63	2523
79	2004
100	1592
126	1264
158	1004
200	798
251	634
316	503
398	400
501	318
631	252
794	200
1000	159



P57 For the inductor $|X_L| = 2\pi f \cdot 10^{-3} \Omega \cdot s$

Frequency	Reactance
(Hz)	(Ohms)
100	1
126	1
158	1
200	1
251	2
316	2
398	3
501	3
631	4
794	5
1000	6
1259	8
1585	10
1995	13
2512	16
3162	20
3981	25
5012	31
6310	40
7943	50
10000	63



Just for fun, here is a plot of the reactance of a 1μF capacitor in series with a 25.3mH inductor.