Physics 122
Chapter 19
Problem Solutions

Q4 You need to ask yourself what "brighter" means. We know about things like charge, voltage, and current but we haven't ever discussed brightness. Suppose you went to the store to buy a dim bulb and a bright bulb. How would you choose? Perhaps the dim bulb would be rated at 15W while the bright bulb would be 100W. Power seems to be the key and we do know how to describe that.

So the question becomes, "In each configuration, which resistance dissipates the most power?" If the two are in series the current in each is the same. Look at Joule's Law expressed in terms of current and resistance.

$$P_1 = I^2 R_1$$
$$P_2 = I^2 R_2$$

Since $R_2 > R_1$ it is clear that for the series combination the larger resistance bulb is brighter.

In the parallel combination the voltage drop across each bulb is the same so we will be well served by Joule's Law expressed in terms of voltage and resistance.

$$P_1 = \frac{V^2}{R_1}$$
$$P_2 = \frac{V^2}{R_2}$$

Now $R_2 > R_1$ implies that the bulb with the larger resistance is dimmer. What is the normal case in your house when you plug in a couple of lamps*?

Q6 The power can be calculated either as that supplied by the batteries or as that dissipated by the resistances. Let's do the latter because it will be easier to think about. As we have information about voltage and resistance we should look at Joule's Law expressed in terms of those two variables.

$$P = \frac{V^2}{R}$$

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* Each outlet delivers 120V – they are wired in parallel. The higher power bulb has the smaller resistance.

To make power larger we should increase the voltage across the total resistance in the circuit and we should decrease the resistance in the circuit. If the batteries are in series their voltages add and we get the largest possible voltage across the resistance. If we connect the resistances in parallel then the resistance will be as small as possible.

Just for fun, let's see how the maximum power compares to the minimum power. For the maximum power $V = V_{Battery} + V_{Battery}$ and $R = \left(1/R + 1/R \right)^{-1}$. If you simplify these and put them into Joule's Law you have

$$P_{Max} = \frac{2V_{Battery}}{R/2} = 4\frac{V_{Battery}}{R}$$

For the minimum power out we could put in the batteries back to back to get a net zero volts out and thus zero power. If we insist that some current flow then the minimum voltage will be achieved by putting the batteries in parallel so that the total voltage across the resistance is just $V_{Battery}$. The largest resistance is made by putting the resistors in series. You should be able to show that this gives one eighth of the power we found for P_{Max}.

Q19 As we discussed in class, during the charging process, current is flowing through the resistor and so some energy extracted from the battery goes to heating the resistor. For those who are interested we can find out how much energy is supplied by the battery and how this is divided between the capacitor and the resistor.

The final energy stored on the capacitor is easy to find as the final voltage across the capacitor is simply the battery voltage.

$$E_{Capacitor} = \frac{C \cdot V_{Battery}^2}{2}$$

The energy supplied by the battery or that dissipated by the resistor is more difficult. If we find one though then the other can be found by addition or subtraction as the energy supplied by the battery is equal to the sum of the energy stored by the capacitor and the energy dissipated by the resistor. Of the two, the battery is easier as the voltage is fixed and so the power supplied is just

$$
\begin{aligned}
P_{Battery} &= I(t)V_{Battery} \\
&= I_0 V_{Battery} e^{-t/\tau} \\
&= \frac{\left(V_{Battery}\right)^2}{R} e^{-t/\tau}
\end{aligned}
$$

To find the energy we have to multiply power at some time by some small Δt and then add those products up for t starting at 0 and t becoming arbitrarily large. Geometrically, this is the V^2/R factor times the area under the exponential curve. It turns out that this area is just τ. Since the time constant is RC, we get the power supplied by the battery is twice that stored on the capacitor. Thus the energy dissipated by the resistor is exactly the same as that stored up on the capacitor.

P6 a) Resistances in series just add up.

(45 + 45 + 45 + 75 + 75 + 75)Ω = 360Ω

b) Add the reciprocals for resistors in parallel. Use the 1/x key on your calculator.

$$
\begin{aligned}
R_P &= \left[\frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{75\Omega} + \frac{1}{75\Omega} + \frac{1}{75\Omega}\right]^{-1} \\
&= \left[0.0222 + 0.0222 + 0.0222 + 0.0133 + 0.0133 + 0.0133\right]^{-1} \Omega \\
&= 9.38\Omega
\end{aligned}
$$

P9 The maximum resistance will be when they are in series.

(680Ω + 940Ω + 1,200Ω) = 2820Ω

The minimum resistance will be when they are all in parallel.

$$
\left[\frac{1}{680\Omega} + \frac{1}{940\Omega} + \frac{1}{1200\Omega}\right]^{-1} = 297\Omega
$$

P16 The question is how to make a resistor smaller by adding something to the circuit. If we think of the water pipe analogy, we want to enable water to flow through an obstruction more easily without actually altering the obstruction. Let's give the water an extra pathway; we should use a second resistor in parallel.

With that said, the problem is mostly done! The equivalent resistance should be 320Ω and one of the parallel resistors is 480Ω. Find the second parallel resistance.

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \tag{P16-1}$$

Solve (P16-1) for, say, R_2. Do the algebra!

$$R_2 = \left(\frac{1}{R_P} - \frac{1}{R_1} \right)^{-1} \tag{P16-2}$$

Put in 320Ω for R_P and 480Ω for R_1.

$$R_2 = 960\Omega$$

P17 You can do this problem with parallel and series combinations of resistors and, I imagine, that is how the author intended for you to do it. After doing that, however, I will also run through a loop analysis do that you can see that it works for simple circuits too.

To use the resistor combination procedure we should first get all three resistors put into one equivalent resistance and find the current supplied by the battery. Then we will see where that can lead us.

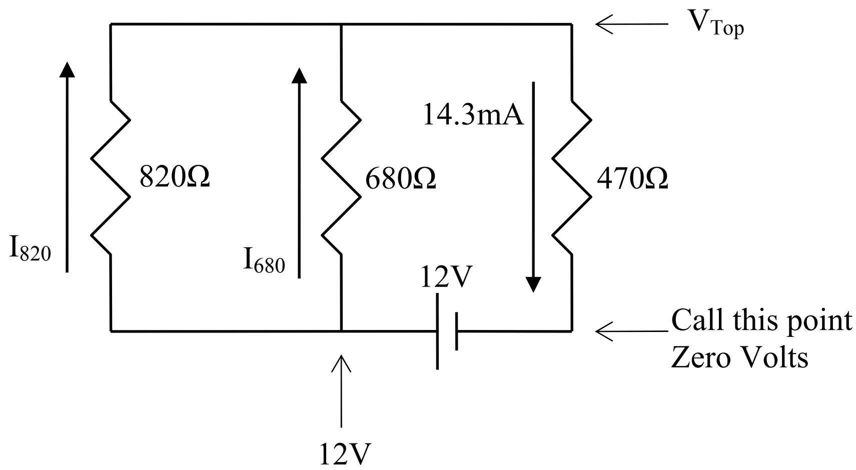
Combine the 820Ω and the 680Ω resistors. They are in parallel.

$$\begin{aligned} R_P &= \left(\frac{1}{820\Omega} + \frac{1}{680\Omega} \right)^{-1} \\ &= 371.7\Omega \end{aligned} \tag{P17-1}$$

Combine the equivalent resistance form (P17-1) with the 470Ω resistor. They are in series.

$$\begin{aligned} R_S &= 371.7\Omega + 470\Omega \\ &= 841.7\Omega \end{aligned} \tag{P17-2}$$

Now Ohm's law gives the current supplied by the battery. $I_{Battery} = 14.3mA$. This is the current through the 470Ω resistor as it is in series with the battery. How can we find the currents through the other two resistors?



To find I_{680} and I_{820} we could use Ohm's law if we knew V_{Top}. To get V_{Top} we can use Ohm's Law and the known current in the 470Ω resistor.

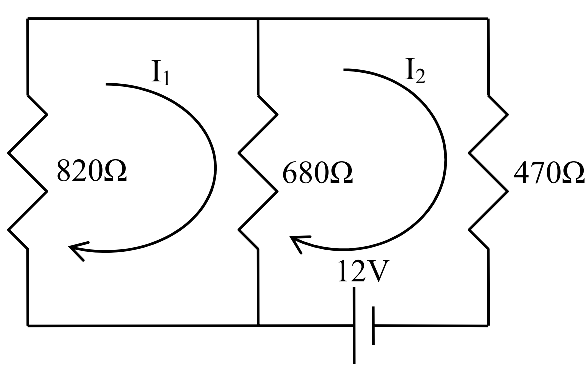
$$\left(0 - V_{Top}\right) = 14.3mA \cdot 470\Omega$$
$$V_{Top} = 6.70V \tag{P17-3}$$

Find I_{820} first using V_{Top} from (P17-3).

$$12V - 6.7V = I_{820} \cdot 820\Omega$$
$$I_{820} = 6.46mA$$

In a similar manner find $I_{690} = 7.79mA$.

How would you do this using Kirchhoff's voltage law?



For loop one we write

$$0 = I_1 \cdot \left(820\Omega + 680\Omega\right) - I_2 \cdot \left(680\Omega\right) \tag{P17-4}$$

And for loop two

$$12V = -I_1 \cdot (680\Omega) + I_2 \cdot (680\Omega + 470\Omega) \qquad \text{(P17-5)}$$

Solve (P17-4) and (P17-5) for I_1 and you get 6.46mA. I_1 is the same thing that we named I_{820} above. The results match. If we use that value for I_1 and solve (P17-4) for I_2 we get 14.3mA. This is what had been called I_{470}. Finally,

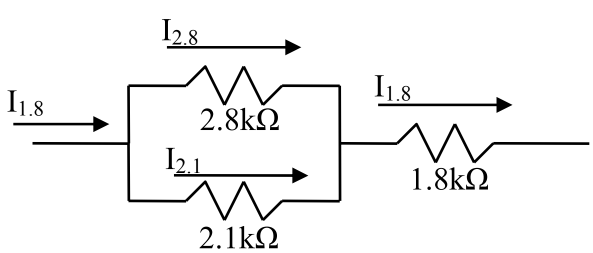
$$I_{680} = I_2 - I_1 \qquad \text{(P17-6)}$$

Notice that I_2 goes through the 680Ω resistor in the same direction as I_{680} and so it must get the same algebraic sign in (P17-6). The subtraction produces 7.79mA as before.

I think the loop analysis is easier but you may not agree. Use the method that appeals to you.

P22 This problem is no longer on the list of assigned problems.
The point of this problem is the thinking process that goes into it. While there are standard techniques for solving resistor network problems† or for solving projectile problems, the same cannot be said for a problem such as this. We have to feel our way through this one without being able to necessarily see all of the details at the outset. So try some of the general strategies that have been helpful in the past.

This problem will be easier to think about if we can look at a picture.



Make the movie in your head. See the current flowing through the wires. See it split to flow through the parallel combination and then recombine. Notice the resistors getting warm as the current flows through them.

Some total amount of current flows through the entire circuit. If I could find out what it is then the applied voltage will be the sum of the voltage drops across either of the parallel resistors and the series resistor. The maximum current will correspond to the maximum voltage. So, what do I know about any of these

† Loop analysis using Kirchhoff's Voltage Law.

resistors? Power is the issue so trot out Joule's Law. I know that I can write down Joule's Law in a way that only involves resistance and current.

$$P = I^2 R \qquad \text{(P22-1)}$$

Do this for each resistor.

$$P_{2.1} = \left(I_{2.1}\right)^2 \cdot 2.1k\Omega \qquad \text{(P22-2)}$$

$$P_{2.8} = \left(I_{2.8}\right)^2 \cdot 2.8k\Omega \qquad \text{(P22-3)}$$

$$P_{1.8} = \left(I_{1.8}\right)^2 \cdot 1.8k\Omega \qquad \text{(P22-4)}$$

Remind yourself of what it is that you are seeking. It is the voltage applied to the resistors when the first one of them reaches a power level of 0.5W.

If I could find out which of these is the largest then I could insist that the currents be such that the largest was just equal to 0.5W. It would seem that I need some information about the relative sizes of the currents. Right away I can see the following.

$$I_{1.8} = I_{2.1} + I_{2.8} \qquad \text{(P22-5)}$$

In addition, because the voltage drop is the same across the parallel resistors I can write

$$\begin{aligned} V_{2.1} &= V_{2.8} \\ I_{2.1}R_{2.1} &= I_{2.8}R_{2.8} \\ I_{2.1}\left(2.1k\Omega\right) &= I_{2.8}\left(2.8k\Omega\right) \\ I_{2.1} \cdot 0.75 &= I_{2.8} \end{aligned} \qquad \text{(P22-6)}$$

Now I can express each of the three currents in terms of $I_{2.8}$ and that will let me compare the power dissipated in each resistor. Use (P22-6) to eliminate $I_{2.1}$ from (P22-5).

$$I_{1.8} = 1.75 \cdot I_{2.8} \qquad \text{(P22-7)}$$

Rewrite (P22-2) and (P22-4) using (P22-6) and (P22-7).

$$\begin{aligned} P_{2.1} &= \left(0.75 \cdot I_{2.8}\right)^2 \cdot 2.1k\Omega \\ &= \left(1.18k\Omega\right)\left(I_{2.8}\right)^2 \end{aligned} \qquad \text{(P22-8)}$$

$$P_{1.8} = (1.75 \cdot I_{2.8})^2 \cdot 1.8k\Omega$$
$$= (5.51k\Omega)(I_{2.8})^2 \quad \text{(P22-9)}$$

Divide (P22-9) by (P22-8).

$$\frac{P_{1.8}}{P_{2.1}} = \frac{5.51}{1.18} \quad \text{(P22-10)}$$

More power is dissipated in $R_{1.8}$ than in $R_{2.1}$. Divide (P22-9) by (P22-3).

$$\frac{P_{1.8}}{P_{2.8}} = \frac{5.51}{2.80} \quad \text{(P22-11)}$$

More power is dissipated in $R_{1.8}$ than in $R_{2.8}$. So the limiting resistor is the 1.8kΩ series resistor. What is the maximum current allowed? Use 0.5W for the power in (P22-4) and solve for the current.

$$0.5W = (I_{1.8})^2 \, 1.8k\Omega$$
$$I_{1.8} = 16.7mA \quad \text{(P22-12)}$$

Use (P22-7) to find $I_{2.8}$.

$$I_{2.8} = 9.52mA \quad \text{(P22-13)}$$

The currents in (P22-12) and (P22-13) enable us to use Ohm's Law to get the voltage drops I need.

$$V_{1.8} = 30.0V$$
$$V_{2.8} = 26.7V$$

The sum of these two, 56.7Volts, is the answer to the problem.

It might be useful to you to review this problem and simply list the major steps in the solution. See if this list makes sense to you. Could you produce such a list on your own?

P23 To compute the sum of the voltage changes around the loop we need to know what each individual voltage change is. Ohm's law will help us if we can find the current. The resistors are all in series, so the total resistance is 22Ω. With a battery voltage of 9Volts we can compute the current as 9/22 A in the counter-clockwise direction.

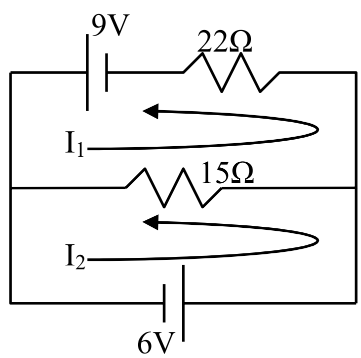
Start at the upper left hand corner and go counter-clockwise.

- The 8Ω resistor gives a (8Ω)(9/22 A) = 36/11 Volt decrease.

- The 12Ω resistor gives a (12Ω)(9/22 A) = 54/11 Volt decrease.
- The 2Ω resistor gives a (2Ω)(9/22 A) = 9/11 Volt decrease.
- The battery gives a 9Volt = 99/11 Volt increase.
- 36 + 54 + 9 = 99

The sum of the voltage changes is zero.

P27 Here is a problem for which a rather mechanical approach will do nicely. Draw the loops, write down the loop equations. Solve the equations for the currents.



For current loop #1

$$9V = I_1 \cdot 37\Omega - I_2 \cdot 15\Omega \qquad \text{(P27-1)}$$

For current loop #2

$$6V = -I_1 \cdot 15\Omega + I_2 \cdot 15\Omega \qquad \text{(P27-2)}$$

If I simply add (P27-1) to (P27-2) the I_2 terms will vanish.

$$\begin{aligned} 15V &= I_1 \cdot 22\Omega \\ I_1 &= 0.68A \end{aligned} \qquad \text{(P27-3)}$$

Use (P27-3) in (P27-2) to get I_2.

$$\begin{aligned} 6V &= -(0.68A) \cdot 15\Omega + I_2 \cdot 15\Omega \\ I_2 &= 1.08A \end{aligned}$$

The current through the 15Ω resistor is 1.08A-0.68A = 0.40A in the direction of I_2. The current through the 22Ω resistor is simply I_1.
Note that you could have said that the 6V battery maintains that voltage difference across the 15Ω resistor and so the current through it must be 6V/15Ω or 0.4A. Similarly, if you follow the outside of the circuit, the two batteries in series maintain 15V across the 22Ω resistor and so the current in that one is 15V/22Ω or 0.68A.

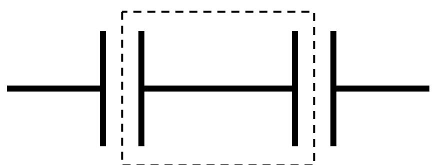
P35 Just remember that capacitors add together in the opposite way that resistors do.

$$C_P = \left[4.7\mu F + 4.7\mu F + 4.7\mu F + 4.7\mu F + 4.7\mu F + 4.7\mu F\right]$$
$$= 28.2\mu F$$

$$C_S = \left[\frac{1}{4.7\mu F} + \frac{1}{4.7\mu F} + \frac{1}{4.7\mu F} + \frac{1}{4.7\mu F} + \frac{1}{4.7\mu F} + \frac{1}{4.7\mu F}\right]^{-1}$$
$$= 0.78\mu F$$

P37 This is much like a circuit where you combine resistors in series and parallel. If we combine the 3.00μF capacitor in series with the 4.00μF capacitor we get an equivalent capacitance of (1/3 + 1/4)$^{-1}$μF = 1.71μF. This can then be combined with the 2.00μF capacitor in parallel. They add, so the total equivalent capacitance is 3.71μF.

P38 The voltage across the 2.00μF capacitor is easy! It is just the 26Volts. What is going on with the other two though? Look at this figure and think about the middle section that consists of one plate from each capacitor and the wire that connects them.



When the capacitors are uncharged this section is clearly electrically neutral. However, even when it is charged up, it must still be neutral because it isn't connected to anything! All that can happen is for charge to move from one capacitor to the other. This means that the two capacitors have exactly the same size charge.

The sum of the voltage changes across the two capacitors must be the 26Volts. So, we can write down the voltage drop across each capacitor and insist on two things. First, they should add to 26Volts. Second, the two charges must be the same.

$$C = \frac{Q}{V}$$

or

$$V = \frac{Q}{C}$$

Use this for each capacitor.

$$V_3 = \frac{Q}{3.00\mu F}$$

$$V_4 = \frac{Q}{4.00\mu F}$$

$$26Volts = V_3 + V_4$$

$$26Volts = \frac{Q}{3.00\mu F} + \frac{Q}{4.00\mu F}$$

$$Q = 44.6\mu C$$

Now use Q to compute V_3 and V_4

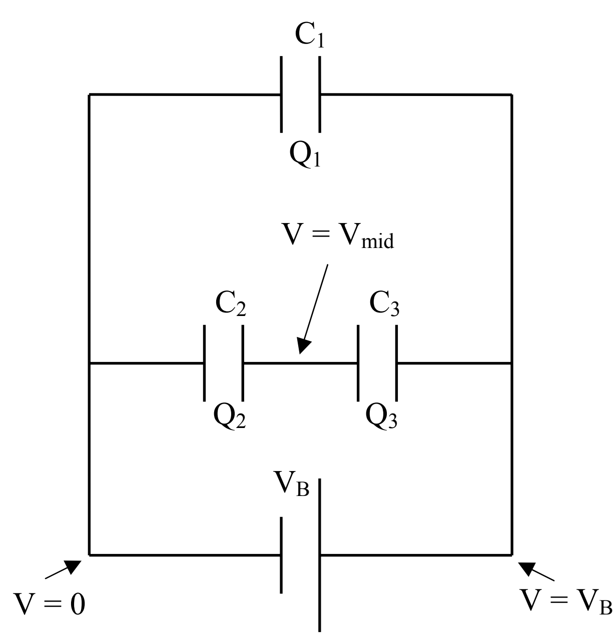
$$V_3 = \frac{44.6\mu C}{3.00\mu F}$$

$$= 14.9Volts$$

$$V_4 = \frac{44.6\mu C}{4.00\mu F}$$

$$= 11.1Volts$$

P42 A picture is again a good place to start. Just get some labels down so that you can describe what is going on.



The fundamental ideas we need to understand this problem are the definition of capacitance,

$$C = \frac{Q}{V}, \tag{P42-1}$$

and conservation of charge that requires

$$Q_2 = Q_3. \tag{P42-2}$$

From looking at the picture I know that the voltage drop across C_1 is V_B. Similarly, the voltage drops across C_2 and C_3 are V_{mid} and V_B - V_{mid} respectively. Write down (P42-1) for each capacitor.

$$C_1 = \frac{Q_1}{V_B} \tag{P42-3}$$

$$C_2 = \frac{Q_2}{V_{mid}} \tag{P42-4}$$

$$C_3 = \frac{Q_3}{V_B - V_{mid}} \tag{P42-5}$$

There are four unknown quantities, the three charges and V_{mid}; the relations (P42-2) through (P42-5) should be a sufficient set to find them. Of course we need only the charges and do not need to solve for V_{mid}. I leave the algebra to you; be sure to do it.

$$\begin{aligned} Q_1 &= C_1 \cdot V_B \\ &= 1.02mC \\ Q_2 = Q_3 &= \frac{V_B}{\frac{1}{C_2} + \frac{1}{C_3}} \\ Q_2 &= 0.34mC \end{aligned}$$

P50 This problem needs only the time dependence of the voltage drop across the resistor. But I did not work that out in class! What are you to do? Slow down and think. How do you always find the voltage drop across a resistor? Ohm's Law. Do you have an expression for the current in an RC circuit? There we go.

$$\begin{aligned} V_R(t) &= \left[I_0 \cdot e^{-\frac{t}{\tau}} \right] \cdot \mathrm{R} \\ &= V_B \cdot e^{-\frac{t}{\tau}} \end{aligned} \tag{P50-1}$$

Of course we know that the time constant, τ, is the product RC. So the first question is answered by a simple division. $C = 2.33 \cdot 10^{-9}$F.

To find the time it takes for the voltage across the resistor to drop to 16V just put in 16V for $V_R(t)$ and solve for t.

$$16V = 24V\left(e^{-t/\tau}\right)$$

$$\ln\left(\frac{2}{3}\right) = -\frac{t}{\tau}$$

$$t = -35\mu s \cdot \ln\left(\frac{2}{3}\right)$$

$$t = 14.2\mu s$$

P51 This is rather like P50! You do not have an expression for the voltage across the capacitor as a function of time. But you do know the charge on the capacitor as a function of time. How is this related to the voltage? C=Q/V or V=Q/C.

$$V(t) = \frac{Q_0}{C}\left(e^{-t/\tau}\right)$$

$$\tau = \left(6.70 \cdot 10^3 \Omega\right)\left(3.00 \cdot 10^{-6} F\right)$$

$$\tau = 2.10 \cdot 10^{-2} s$$

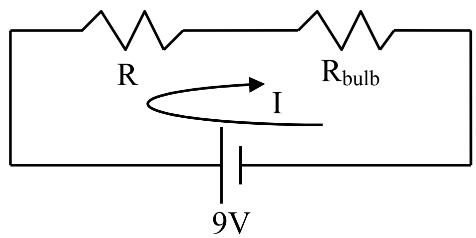
The initial voltage is Q_0/C and we want the time such that V(t) is 0.01 times that.

$$(0.01)\frac{Q_0}{C} = \frac{Q_0}{C}\left(e^{-t/\tau}\right)$$

$$\ln(0.01) = -\frac{t}{0.021s}$$

$$t = 0.097s$$

P87 My translation of the words in this problem statement is that we want to have a 3V drop across the light bulb. Would you agree that mine is a sensible reading? Let's proceed along that line anyway. Here is a picture with the light bulb represented by the resistor R_{bulb}.



I know that the voltage drop across R should be 6V (Why[‡]?). If I had the current I could use Ohm's Law to find the value of R. How can I find the current? I am told that 3V applied to R_{bulb} will result in 2.5W being dissipated. Use Joule's Law in the form that contains current and voltage.

$$P = IV$$

$$2.5W = I \cdot 3V$$

$$I = \frac{5}{6}A$$

Now trot out Mr. Ohm and finish the problem. R = 7.2Ω.

[‡] The sum of the voltage drops around the loop must be zero. A 9V rise at the battery and a 3V drop at the light bulb leaves a 6V drop for the resistance R.