Physics 122
Chapter 18
Problem Solutions

Q9 This question is an excellent illustration of the need to carefully formulate the questions you ask. Relating to the first expression, $P = V^2/R$ the problem statement reads "…indicates that the power dissipated in a resistor decreases if the resistance is increased,…" The expression does not say that. The expression says that "*If the voltage is held fixed*, then the power dissipated in a resistor decreases as the resistance is increased."

With that as a hint, reexamine the question and see if you can understand what is going on and why the two forms of Joule's Law ($P = V^2/R$ and $P = I^2R$) are not contradictory.

In the second of the two expressions there is an assumption that the current is held constant. This is quite different from holding the voltage constant. What you must do is make your assumption explicit and then examine the expression to see what it predicts.

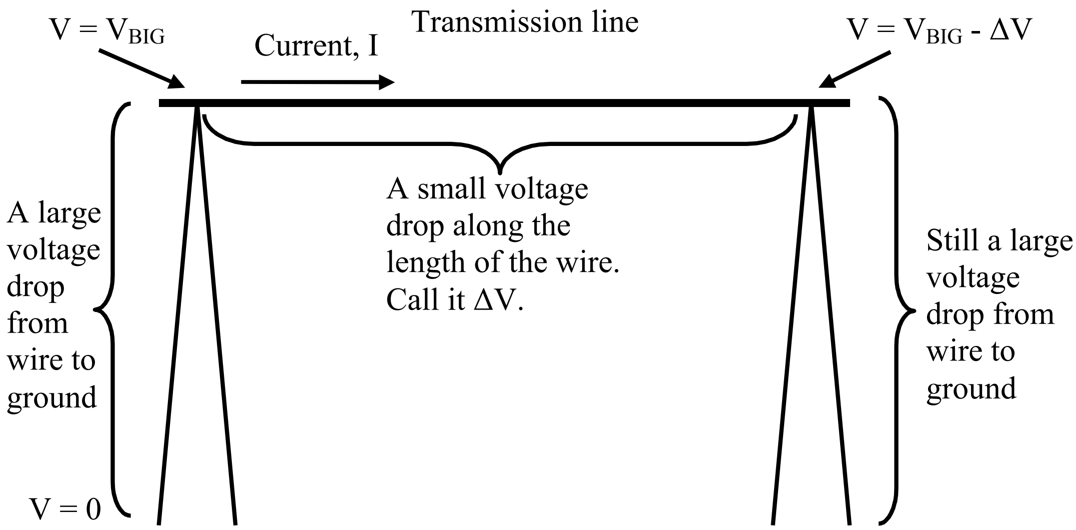
Assume that the current is held constant. What does $P = V^2/R$ predict will happen when the resistance is increased? To answer this you have to know what the voltage will do if the current is held constant. Ohm's law tells you this; $V = IR$. In this case the voltage is proportional to the resistance and so voltage squared is proportional to the square of the resistance. Thus, the prediction is that the power will be proportional to the resistance and so the power dissipated will increase as the resistance increases. This, of course, matches with the prediction from $P = I^2R$ as Ohm's law is the device you need to convert either of these expressions to the other.

Q13 The constant in the transmission of power in the high voltage lines is the wire itself. That means that the resistance over some set distance is fixed. Now I want to pose the following question, "For a fixed resistance and a fixed <u>transmitted</u> power, will an increase in the voltage of the line increase or reduce the <u>wasted</u> power?" The picture below might help make this clear.

Let the power transmitted be P_T. This is a fixed quantity.

$$P_T = I \cdot (V_{BIG} - \Delta V)$$
$$P_T \simeq I \cdot V_{BIG} \qquad \text{(Q13-1)}$$

The second line is approximately true because ΔV is small compared to V_{BIG}.



The wasted power can be written as

$$P_W = I^2 R \qquad \text{(Q13-2)}$$

The resistance is fixed by the wire itself and we can get the current from (Q13-1). Now we have the wasted power in terms of things that are constant and the transmission voltage.

$$P_W = \left(\frac{P_T}{V_{BIG}}\right)^2 \cdot R \qquad \text{(Q13-3)}$$

This expression makes it clear that we should increase the transmission voltage to reduce the wasted power. This is an approximation but, as the transmission voltage will be something like 10^5V and the voltage drop along the wires would be only some hundreds of volts, it is a very good approximation.

Q18 Current is charge in motion. For it to be used up the charge itself would have to, in some way, disappear. But you cannot destroy or, for that matter, create the charge. Think of this in terms of our water in the pipes analogy. The resistor is a plug of steel wool in the pipe. The water flows through it. The water does not vanish or get stored up. At any moment the water flows out of the plug at exactly the same rate it flows in. However, a force is required to make it move through the steel wool; this force derives from the difference in pressure on the two sides. On the outlet side the pressure is reduced. In the case of the current flowing through the resistor the voltage on the outlet side is reduced – there is a voltage drop across the resistor.

P4 Use Ohm's law. $R = \frac{V}{I}$ Inserting the given values, we find $\frac{120V}{4.2A} = 28.6\Omega$.

P7 The current is found with Ohm's law.

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{240Volts}{9.6\Omega} \\ &= 25A \end{aligned} \tag{P7-1}$$

To find the charge that passes through in 50min you need to recall that current is simply the charge per unit time that passes a given point. 1A is 1C per second. Use the current found in (P7-1).

$$\begin{aligned} Q &= I \cdot \Delta t \\ &= 25A \cdot 50\min \frac{60s}{\min} \\ &= 7.5 \cdot 10^4 C \end{aligned}$$

P11 The first part of this problem is much like P7; you have two out of three of the quantities related by Ohm's law and you are to find the third.

$$\begin{aligned} R &= \frac{V}{I} \\ &= \frac{12Volts}{0.6A} \\ R &= 20\Omega \end{aligned}$$

Joule's law is related to the energy lost in the resistor in that power is the rate at which energy is lost. So, as the rate would be a constant, the energy loss is simply rate multiplied by time. Because I have all three of current, voltage, and resistance I can choose any form of Joule's law I prefer.

$$P = IV$$

$$\text{Energy Lost} = P \cdot \Delta t$$

$$\text{Energy Lost} = (0.6A)(12V)(60s)$$

$$\text{Energy Lost} = 432J$$

P12 Our model for resistance is $R = \frac{\rho L}{A}$. The resistivity we look up on page 501. Recall that the area of a circle is $\frac{\pi d^2}{4}$. Solving for the diameter squared we get $d^2 = \frac{4\rho L}{\pi R}$. Insert our values. $d^2 = \frac{4(5.6 \cdot 10^{-8}\,\Omega m)(1.00m)}{\pi(0.32\Omega)}$ The diameter is $4.7 \cdot 10^{-4}$m or about a half a millimeter.

P28 Joule's law in the form $P = V^2/R$ is useful here. Solve for the voltage. $V = \sqrt{(0.25W)(2.7 \cdot 10^3\Omega)}$. V = 26.0Volts.

P34 This is one step more than P11. We need to find the energy used in a resistance (the light bulb) and then multiply the total energy by the cost per unit energy to find the total cost.

There are 24·365 = 8760 hours in a year. A 25W bulb is the same as a 0.025kW bulb so it will use 219kWh[*] of energy over the course of the year. At $0.095 per kWh the total bill is $20.81.

P40 Let's ask ourselves a few questions to help get this problem started.

- What is going on here? Water is getting heated.
- What is heating the water? The electromagnet.
- Why is the electromagnet getting hot? It has electrical current passing through it and there is some resistance.
- So what is the essential connection of cause and effect? The current produces heat (Joule's law) and that heat is absorbed by the water (something like calorimetry.)

Now I think I see how to work this problem. Joule's law will give me a rate of heat production[†] and I will have to set this equal to a rate of heat removal by the water. Let's find the rate of heat production first.

---

[*] 1kWh = 1000J/s·3600s = 3.6MJ

[†] This is power.

$$P_{in} = I \cdot V$$

$$P_{in} = (17.5A)(240Volts)$$
$$= 4200W \qquad \text{(P40-1)}$$

That wasn't too hard. How about the removal of that energy? When you add heat to water its temperature goes up. In this case it goes up by ΔT = 7.5ºC. The connection between heat and temperature change is the heat capacity.

$$m \cdot c = \frac{Q_{in}}{\Delta T} \qquad \text{(P40-2)}$$

We are looking for the rate of water flow, mass per unit time. If (P40-2) is solved for the mass of the water and then divided by the time it takes that water to pass through the magnet we will have the quantity we seek.

$$\frac{m}{\Delta t} = \frac{1}{c \cdot \Delta t} \cdot \frac{Q_{in}}{\Delta T}$$

$$\frac{m}{\Delta t} = \left(\frac{Q_{in}}{\Delta t}\right)\left(\frac{1}{c \cdot \Delta T}\right) \qquad \text{(P40-3)}$$

The first term on the right hand side in (P40-3) is the rate of heat added to the water by the magnet. But this must be the same as what the magnet puts out – the quantity we found in (P40-1).

$$\frac{m}{\Delta t} = (4200W)\left(\frac{1}{(4186J/kg°C)(7.5°C)}\right)$$
$$= 0.134kg/s$$

P49 In the microscopic picture, what is current? It is a bunch of electrons passing by. The size of the current tells us the number of electrons that pass by per unit time. Clearly this must be in some way related to the speed with which the electrons actually move. Perhaps we may begin with just this one small piece though. Convert 2.3μA into electrons per second.

$$2.3 \cdot 10^{-6} A = (2.3 \cdot 10^{-6} C/s)\left(\frac{1 \text{ electron}}{1.6 \cdot 10^{-19} C}\right)$$
$$= 1.44 \cdot 10^{13} \text{electrons/second} \qquad \text{(P49-1)}$$

Think of the water in the pipes analogy. This would be some number of water molecules per second that pass by in the pipe. If we wanted the speed of the water, what would we need to know? Suppose that there were two pipes – one skinny and one fat – through which the water could flow. If the number of molecules per second were fixed, would the water speeds be the same? No. The water would have to move more quickly in the skinny pipe. We will have to know the size of the wire to answer this question.

We are in luck! The diameter of the wire is given to us. What else do we need to know? Thinking again of the water analogy, consider two types of water flowing through the pipe. The first is nice and clear with no bubbles of air but the second is all frothy as if it just passed through an aerator. If the material mover past us at the same speed in each case will we count passing water molecules at the same rate? No. There is more space between the molecules in the frothy water and, for any unit volume of water that pass by, the frothy water contains fewer molecules. We need to know the number of electrons per unit volume that are free to move in copper wire.

To progress from here we must realize that one electron per copper atom is free to move in the wire. This was given in the text of our chapter‡. Then the density of copper atoms in the wire will be the same as the density of the free electrons. One mol of Cu has a mass of 63.5g. The density of Cu is $8.9 g/cm^3$. Thus there are

$$\begin{aligned} \mathrm{mol/cm^3} &= \frac{\mathrm{g/cm^3}}{\mathrm{g/mol}} \\ &= \frac{8.9\mathrm{g/cm^3}}{63.5\mathrm{g/mol}} \\ &= 0.14\, mol/cm^3 \end{aligned} \tag{P49-2}$$

Looking up Avogadro's number we convert this to $8.43\cdot10^{22}$ electrons/cm^3. From (P49-1) we know that $1.44\cdot10^{13}$ electrons must pass by per second. What volume does this correspond to? Number of electrons = (volume)(density).

$$\begin{aligned} \mathrm{Volume} &= \frac{1.44\cdot10^{13}\, electrons}{8.43\cdot10^{22}\, electorns/cm^3} \\ &= 1.71\cdot10^{-10}\, cm^3 \end{aligned} \tag{P49-3}$$

A tube of electrons with the given diameter and this volume must pass by in one second. If I find the length of that tube then I will know the speed of the electrons because an electron in the very back of the tube will just cover that distance in one second.

‡ page 509

$$\text{Volume} = (\text{cross sectional area of tube})(\text{length of tube})$$

$$1.71\cdot 10^{-10}\,cm^3 = \pi\left(\frac{0.065cm}{2}\right)^2 (L)$$

$$L = \frac{2\cdot 1.71\cdot 10^{-10}\,cm^3}{\pi\left(0.0325cm\right)^2} \qquad \text{(P49-4)}$$

$$= 5.2\cdot 10^{-8}\,cm$$

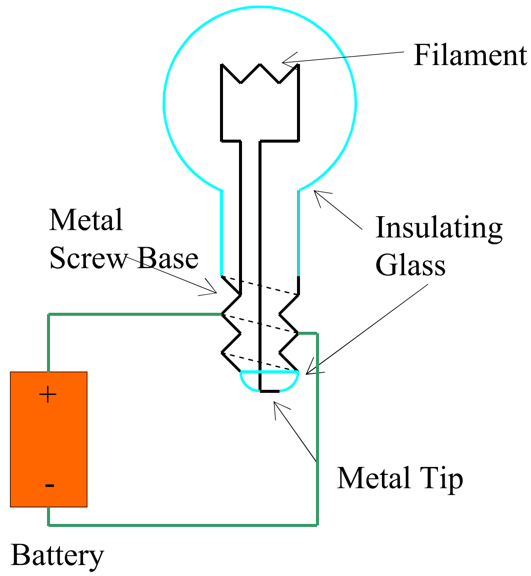
And so the drift speed of the electrons is $5.2\cdot 10^{-8}$cm/s. Mighty SLOW for something as quick as an electrical current! As we will see, the electric field propagates at the speed of light (variable for different materials but always really fast) and it is that which pushes the electrons along. So when you flip the light switch the electrons in the entire circuit get pushed on pretty much instantly.

P63 Remember our model for resistance. $R = \frac{\rho L}{A}$ This problem asks up to reduce the length by a factor of 2. That is we are to replace L by L/2. Then we put the wire pieces side by side so that the area is increased by a factor of 2. We replace A by 2A. Each of these replacements puts a 2 into the denominator of the fraction. The result is a number ¼ as big as what we had before.

P68 This is a lot like P63. If we can figure out what happens to L and A we are all set. "Three times as long" must mean that L is replaced by 3L. But what do we do with A? The key here is to think of what does not change. The amount of metal in the wire stays the same. How to measure that? You could ask that its mass remain fixed but something that is the equivalent of that (assuming that the density does not change) is to say that the volume of metal stays the same. Volume is Area times Length. If L increases by a factor of 3 then A must decrease by a factor of 3 is their product is to remain unchanged. The area A is replaced by A/3. Each of these changes puts a 3 into the numerator of the fraction. The resistance will increase to 9 times its former value or 9.0Ω.

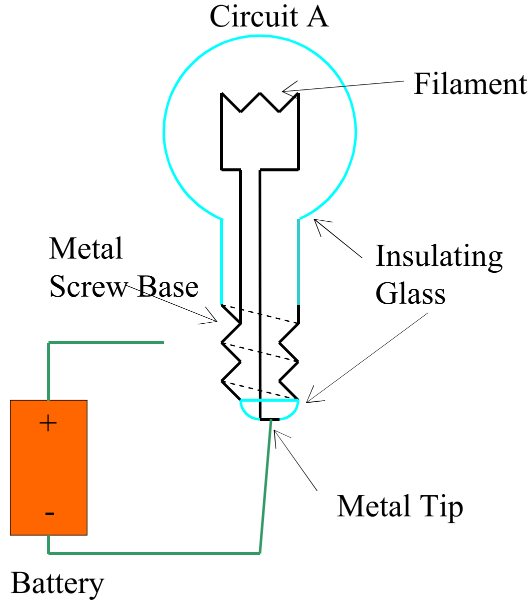
Additional Problem

A1 In which of the circuits shown below will the light bulb light up?



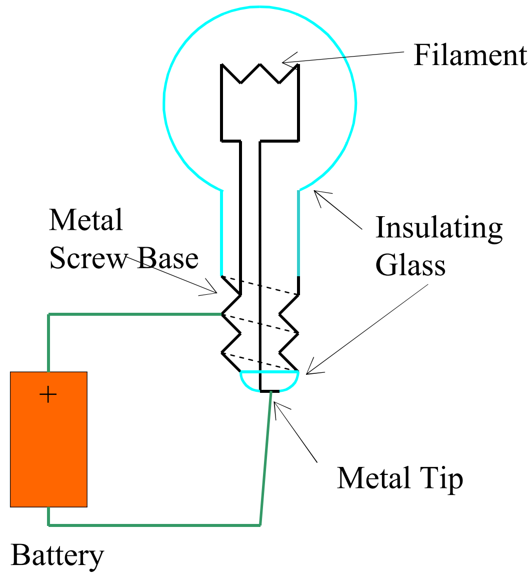
The bulb does not light. There is no return path for current which passes through the filament.

Circuit A



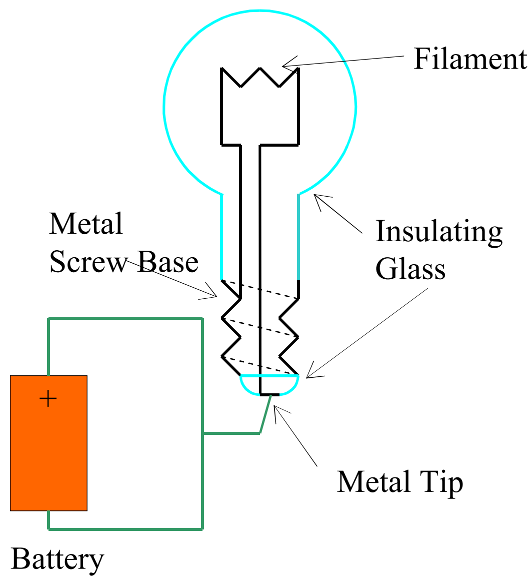
Circuit B

The bulb does not light up. The upper wire does not connect to the bulb.



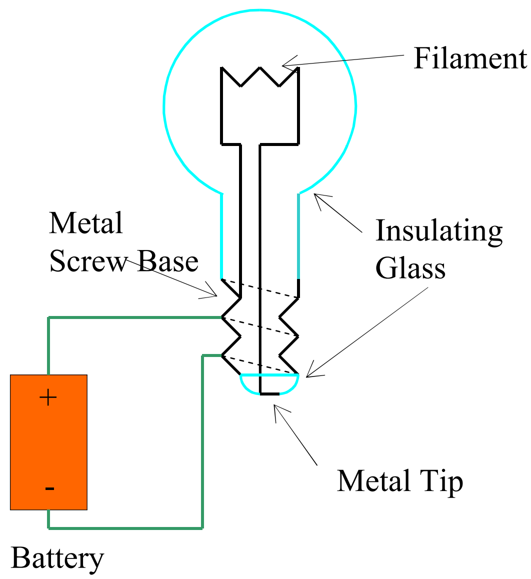
The bulb lights. Current passes out of the battery, through the filament, and then back to the battery.

Circuit C



Circuit D

The bulb does not light for the same reason as in circuit A.



Circuit E

The bulb does not light. Compare this to circuits A and D.