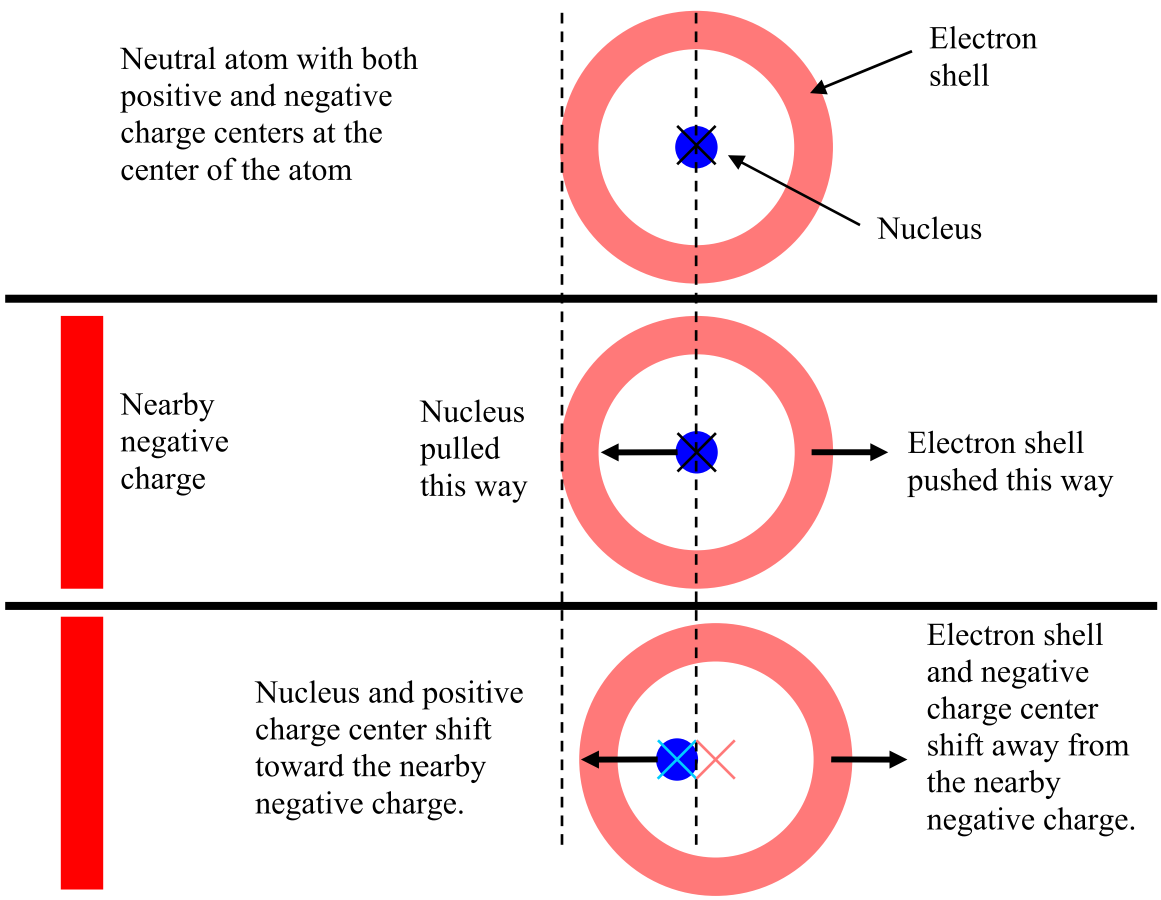
Physics 122
Chapter 16
Problem Solutions

Q4 All charges exert forces on all other charges. The fact that an object happens to be electrically neutral does not change this. We tend to think of the neutral object as being without charge but this is not true. It is composed of atoms all of which are in turn composed of protons, neutrons, and electrons.

Suppose that we bring a negative charge near to a neutral object. The electrons will be repelled, the protons will be attracted, and the neutrons will not be affected. If each atom were spherically symmetric and remained that way in the presence of the negative charge then the attractive and repulsive forces would add to zero and there would be no net attraction. But atoms are not perfectly rigid and so this is not the case.

Consider a single atom in the neutral object and how it reacts to the nearby negative charge.



Because the electrostatic force gets stronger with decreasing distance, the attractive force between the dissimilar charges is now larger than the repulsive force between the like charges. It is true that the difference in the two distances is a very small amount and that the force on any one atom is extremely small. However, there are many atoms in even the smallest piece of dust and the resulting total force can easily be greater than the weight of a small object.

Note that it makes no difference to the outcome if we bring a positive rather than a negative charge near to the atom. It is still attracted. Be sure that you think this through and that it makes good sense to you. If it does not, come talk with me.

Q6 The short answer to this problem is that the free charges are those that are present in a metal when it is neutral and are free to move about the entire piece of metal. The net charge is the extra charge you add to or remove from the metal. To consider their relative sizes we must work a little harder.

Consider a typical conductor such as aluminum or copper. Each metal atom has the same number of electrons as protons and so the metal as a whole is electrically neutral. Because of the way metal atoms bond to form the solid each atom will have one (Al) or two (Cu) of its electrons not confined to the immediate vicinity of the nucleus with which it was associated. It or they are free to move about the entire solid piece. Thus the density of conduction electrons[*] is about the same as the density of the metal atoms. In the case of Al this is about one mol of conduction electrons in $10cm^3$ of metal.

Suppose that I want to add[†] some additional electrons to my $10cm^3$ piece of Al. These additional electrons would constitute a charge imbalance or a net charge. Of course I could add just one extra electron and this would be the smallest net charge I could have on my metal piece. What is the largest net charge I could actually hope to put on a $10cm^3$ sphere[‡] of Al? As I put more and more charge on the metal, the electric field at the surface gets bigger and bigger. At some point the force it exerts on the O_2 molecules in the adjacent air is enough tear them apart. The air becomes charged and as the opposite charges flow to the metal its net charge is reduced. The biggest electric field you can have before breaking down dry air is about $3{\cdot}10^6$V/m. This will be the size of the electric field at the surface of my $10cm^3$ sphere when I have about $3{\cdot}10^{-4}$C of net charge. There are about 10^5 C in a mol of charge so net charges will typically be something like one billion (or more) times smaller than the free charge.

[*] The problem statement refers to the conduction electrons as free charge – that is free to move.
[†] I could also remove electrons to leave the Al with a net positive charge.
[‡] Sharp curves, edges, and points lead to discharge with smaller net charges. A sphere will enable the largest charge to be stored before the air begins to break down and the charge is allowed to leak off.

Q12 See Q6 for the reason the bits of paper are attracted in the first place. The net charge on the ruler is sitting on its surface. These charges are not free to move on the ruler because it is an insulator but they can adhere to some other object that happens to touch them. Should some of the charge on the ruler transfer itself to the small paper piece, then that bit of paper will no longer be neutral – it now has the same charge as the ruler. It is true that the bulk of the paper piece is still polarized and attracted to the ruler but that interaction is much weaker than the force between the two unbalanced charges.[§] Because these two charges are the same sign, the paper bit is repelled from the ruler and it flies away.

Q20 We are trying to add two vectors (the electric field from each charge) and get zero. For this to happen, the two vectors must be opposite in direction and of the same magnitude. Consider first the case of opposite charges. This figure shows the direction of the field from each charge at several points in space.

Recall that the electric field points away from positive charge (red) and toward negative charge (blue). Between the charges the two fields both point to the right. Thus they cannot add to zero there. To the left of the charges, any point is closer to the larger size charge. At a given point in space the field will have a larger magnitude if the point is closer to the charge and a larger magnitude if the size of the charge is greater. For any point in this region of space the field from the positive charge will therefore be larger in magnitude than the field from the negative charge. Complete cancellation of the fields cannot happen here either.

To the right of the charges we have points that are closer to the smaller charge and could therefore have the same magnitude. They are also pointing in opposite directions. Using Coulomb's Law and the principle of superposition we can write down the combined electric field in this region. Let's call the direction to the right the positive x direction.

$$\vec{E} = \frac{k(2Q)}{r_2^2}\hat{x} + \frac{k(-Q)}{r_1^2}\hat{x} \qquad \text{(Q20-1)}$$

Set the total field to be zero and solve (Q20-1) for the relationship between r_2 (the distance from the 2Q charge) and r_1 (the distance form the –Q charge).

[§] See P35 and P40.

$$\frac{r_2^2}{2} = r_1^2$$

$$r_2 = r_1\sqrt{2} \qquad \text{(Q20-2)}$$

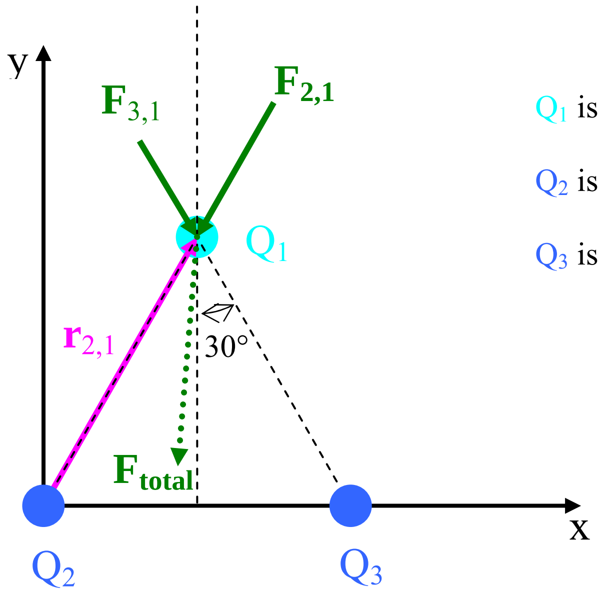
I have not put in all of the algebraic manipulations needed to get from equation Q20-1 to the result in equation Q20-2. You should put these steps in yourself. This is important. You will learn far more by actually moving your pencil and writing the steps down than you will by simply reading over one of my solutions. If you have printed this off, there is plenty of space to add the missing algebra on these pages. Make a habit of doing this. You should write in your textbook too! Underlining and highlighting don't count; write in missing steps, questions, and restatements that make sense to you. Writing makes your brain work in a way that just reading does not.

Note that (Q20-2) does not tell you where the field will be zero. However we are not stuck yet. We also know that the +2Q charge is a distance *l* farther away from the field point than is the –Q charge.

$$r_2 = l + r_1 \qquad \text{(Q20-3)}$$

The two relations (Q20-2) and (Q20-3) constitute a system of two equations in two unknowns. Solve them to find that $r_1 = \frac{l}{\sqrt{2}-1}$.

P17 As is nearly always the case, you should have a picture with a coordinate system drawn before you write down any equations. If this was not the case for you, stop reading this and go make a good diagram. That might be enough to get you going on the solution.



Q_1 is $+4\cdot10^{-6}$C at (0.6m, 1.04m)

Q_2 is $-8\cdot10^{-6}$C at (0m, 0m)

Q_3 is $-6\cdot10^{-6}$C at (1.2m, 0m)

I put Q_2 at the origin and Q_3 on the positive x axis. Another sensible choice would have Q_1 at the origin as we are only looking for the forces acting on that charge. The final result would be unaffected by a shift in the location of the origin – why is that?[**]

Notice that I have drawn the force vectors with some care. I know that opposite charges attract and so the vectors are drawn in that sense. I also know that, other things being equal, larger charges exert larger forces; the lengths of the vectors reflect this as well. Why should I do this? My pictures tell me how to proceed with writing down a set of equations that will yield a quantitative result. That result will then be interpreted in the context of the drawing. If the drawing is carefully done it will help me to see meanings and to spot errors. If you will invest time and energy at this point you will save in the long run and your learning will be more efficient.

I am asked for the net force on Q_1. The principle of superposition and the drawing tell me what to write. I do not need Coulomb's law at this point.

$$\vec{F}_{total} = \vec{F}_{2,1} + \vec{F}_{3,1} \qquad \text{(P17-1)}$$

Equation (P17-1) tells me what I need to do next; to find the total force I need to write down the individual forces $\vec{F}_{1,2}$ and $\vec{F}_{1,3}$. Now I need Coulomb's Law and my skills at working with vectors.

[**] The answer is a force which has a size and a direction only. There is no reference to a location in space. If we use a new set of coordinate axies that are not rotated then we do not change our names for directions in space.

For each force I have a series of steps to get through.

- Find the distance between the charges.
  - This is easy here because I was told that it is 1.2m
- Calculate the magnitude of the force using Coulomb's law.

$$F_{2,1} = \left| \left(\frac{kQ_1Q_2}{\left(r_{2,1}\right)^2} \right) \right| \tag{P17-2}$$

$$F_{2,1} = \left| \left(\frac{\left(9.0\cdot10^9\, Nm^2/C^2\right)\left(-8.0\cdot10^{-6}C\right)\left(4.0\cdot10^{-6}C\right)}{\left(1.2m\right)^2} \right) \right| \tag{P17-3}$$

$$= 0.20N$$

  - Note the absolute value signs! I am just trying to find the size of the force. We will get the direction right in a minute.
- Using the directions I set up in my drawing, I find the size of the components of the force.
  - This is an equilateral triangle, so the angle between the x axis and the left side of the triangle is 60º.

$$\begin{aligned} F_{2,1,y} &= F_{2,1}\cdot\sin\left(60°\right) \\ &= 0.20N\cdot\sin\left(60°\right) \\ &= 0.17N \\ F_{2,1,x} &= F_{2,1}\cdot\cos\left(60°\right) \\ &= 0.20N\cdot\cos\left(60°\right) \\ &= 0.10N \end{aligned} \tag{P17-4}$$

- Get the direction right for each component. The force is down and to the left. In the coordinate system we used for this problem that is the negative direction for both components. I will put vector arrows over the force symbols so that I know that I have the direction included now.

$$\begin{aligned} \vec{F}_{2,1,y} &= -0.17N \\ \vec{F}_{2,1,x} &= -0.10N \end{aligned} \tag{P17-5}$$

Now I repeat the process for the force on charge 1 due to charge 3. Try it out; see if you get this.

$$\begin{aligned} \vec{F}_{3,1,y} &= -0.13N \\ \vec{F}_{3,1,x} &= 0.08N \end{aligned} \tag{P17-6}$$

The last step in this problem is an easy one. Add the x components to get a total x component. Add the y components to get a total y component. NOW STOP! Giving a vector in term of its components gives all of the information that there is. Finding the magnitude and direction is not a superior answer. I will supply it here as the problem statement asks for it. However, I will always be quite happy to have a solution given to me in the form of vector components. Keep this in mind when you take exams!

Simply add the results in (P17-5) and (P17-6) to get the following.

$$\vec{F}_{total,1} = \left(-0.02N\right)\hat{x} + \left(-0.30N\right)\hat{y}$$

$$\left|\vec{F}_{total,1}\right| = 0.30N \qquad \text{(P17-7)}$$

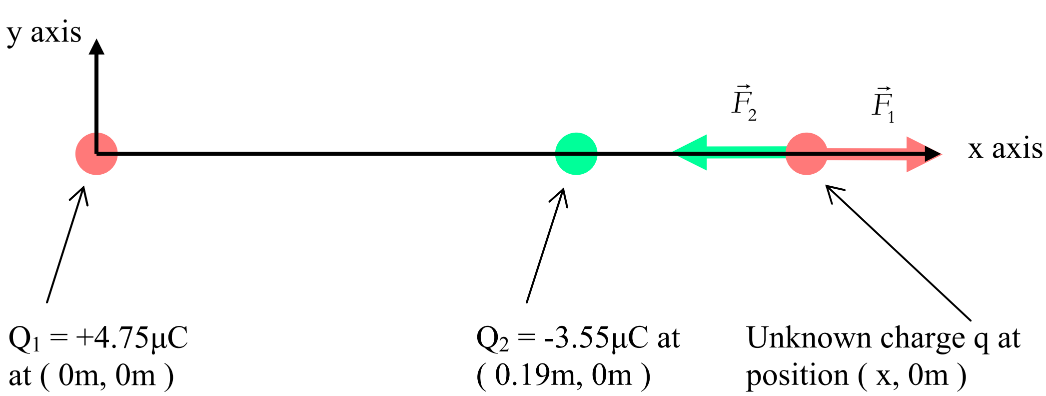
$$\arctan\left(\frac{-0.30N}{-0.02N}\right) = 89°$$

Notice that we now have to interpret this 89º. It is not 89º in the positive direction from the x axis. It is 89º from the negative x axis in the positive (counter-clockwise direction).

P20 Before you proceed further with this problem you should be sure that you completely understand Q20. If you don't, then go review that solution and then think about this problem again.

What is the first thing to do? Have you done it? If you haven't, stop reading this and draw a useful picture.

We are to have two known charges placed in a fixed orientation and then we will be examining the net force on a third charge of unknown size and location. I will choose a coordinate system that is as convenient as possible. The +4.75μC charge will be at the origin and the -3.55μC will be on the x axis at 0.19m. The unknown charge will (on the basis of arguments given in Q20) be located on the x axis at a distance greater than 0.19m. It will have a charge q and a position x.



In this drawing I made an assumption about q. What was it and does it matter?[††]

I want the total force, $\vec{F}_1 + \vec{F}_2$, to be zero. So, I have to write down each, add them, and insist that the vector sum be zero. Let's trot out Coulomb's law for the first charge.

$$\vec{F}_1 = \left(\frac{kQ_1 q}{x^2}\right)\hat{x} \quad \text{(P20-1)}$$

Repeat for $\vec{F}_2$. You should fill in all of the details that lead to the next result. If you just read over this solution and say to yourself that you understand it then you will miss the most important opportunity for learning that this class offers. Make your hand do the work – it is the only way you will actually create the knowledge in your mind[‡‡].

$$\vec{F}_2 = \left(\frac{kQ_2 q}{\left(x - 0.19\text{m}\right)^2}\right)\hat{x} \quad \text{(P20-2)}$$

Now add (P20-1) to (P20-2) and set the sum equal to zero.

$$\begin{aligned} 0 &= \left(\frac{kQ_1 q}{x^2}\right)\hat{x} + \left(\frac{kQ_2 q}{\left(x - 0.19\text{m}\right)^2}\right)\hat{x} \\ 0 &= \left(\left(\frac{Q_1}{x^2} + \frac{Q_2}{\left(x - 0.19\text{m}\right)^2}\right)\left(kq\right)\right)\hat{x} \end{aligned} \quad \text{(P20-3)}$$

The only way for (P20-3) to be true is for at least one of the two factors to be zero. Neither k nor q[§§] can be zero so it must be the case that the first factor is zero. Write this down and solve for x.

[††] I assumed q to be positive. If q were to be negative then the direction of each force would be reversed. As I want them to add to zero all that matters is they point in opposite directions. Another way of thinking about this is that the total electric field at this point will be zero (Q20!) and the force on q will be $q \cdot \vec{E}$.

[‡‡] Knowledge in *my* mind does not do *you* any good!

[§§] If q is zero then the problem becomes trivial and uninteresting; any location will work!

$$0 = \frac{Q_1}{x^2} + \frac{Q_2}{(x - 0.19\text{m})^2}$$

$$-\frac{Q_1}{Q_2} = \frac{x^2}{(x - 0.19\text{m})^2}$$

$$\sqrt{-\frac{Q_1}{Q_2}} = \pm\frac{x}{(x - 0.19\text{m})}$$

$$\sqrt{-\frac{Q_1}{Q_2}}(x - 0.19\text{m}) = \pm x$$

$$(-0.19\text{m})\sqrt{-\frac{Q_1}{Q_2}} = x\left(-\sqrt{-\frac{Q_1}{Q_2}} \pm 1\right)$$

$$\frac{(-0.19\text{m})\sqrt{-\frac{Q_1}{Q_2}}}{\left(-\sqrt{-\frac{Q_1}{Q_2}} \pm 1\right)} = x$$

We require that q be to the right of Q_2. Put in the values for the charges and work out our solutions.

$$x = 1.37m \qquad \text{(P20-4)}$$

or

$$x = 0.10m \qquad \text{(P20-5)}$$

Only the first solution satisfies our requirement that q be to the right of Q_2. Where does the other solution come from? Equation (P20-2) has built into it the assumption that $x > 0.19$m. When we solve for x to get (P20-4) and (P20-5) we are only finding x so that the ratio of the distances from Q_1 and Q_2 are correct. We have thrown away the information about direction. For a point x between Q_1 and Q_2 we must rewrite (P20-2). Note how our drawing has not only told us how to write down the appropriate expressions but it has also told us how to interpret the results of our calculations.

P27 If someone asks you about the acceleration of some object you should ask yourself why the object is accelerating at all. The answer is always the same – there is some net force acting upon it. That is what Newton's 2nd law is all about. This will also tell you how to proceed at least some way toward the answer to the question.

$$\vec{a} = \frac{\vec{F}_{net}}{m} \tag{P27-1}$$

So what is exerting a force on the electron? The electric field is and there is nothing else around. Suppose that the electric field in is the positive x direction. The force is $q \cdot \vec{E}$ or

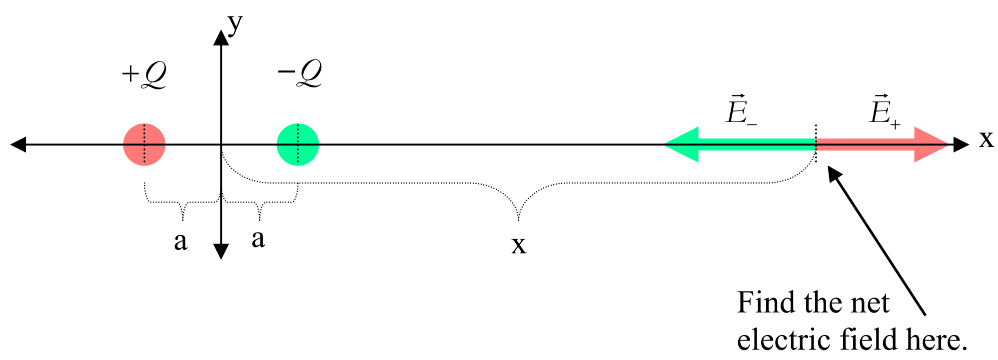
$$\begin{aligned} \vec{F} &= \left(-1.60 \cdot 10^{-19}\,\mathrm{C}\right)\left(\left(750\mathrm{N}/\mathrm{C}\right)\hat{x}\right) \\ &= -\left(1.20 \cdot 10^{-16}\,\mathrm{N}\right)\hat{x} \end{aligned} \tag{P27-2}$$

The force is in the opposite direction to that of the electric field; this is because the charge on an electron is negative. Combine (P27-1) and (P27-2) to find the acceleration of the electron.

$$\vec{a} = \left(-1.32 \cdot 10^{14}\,\mathrm{m}/\mathrm{s}^2\right)\hat{x}$$

Note two things about this answer. First, the acceleration is a very large number. In the event of using such a field to accelerate an electron, the electron would reach speeds near to that of light in a short time and a small distance. As we will see at the end of the semester a correct description of motion at these speeds requires a reformulation of our ideas of time and space. Second, the acceleration is also in the direction opposite to the field.

P35 This problem is closely related to Q20 and P20. The reason we are interested in this particular setup is that atoms are, as a rule, electrically neutral but if they are put into a region of space where an electric field is present the positive and negative charge centers will shift in opposite directions; then the atom will look like this. As we will find out, light is an electromagnetic wave and so any illuminated atom will experience this. We proceed as in P20 but now the origin is in the middle of the two charges.



Call the electric field due to the positive charge $\vec{E}_+$ and that from the negative charge $\vec{E}_-$. The total electric field $\vec{E}_T$ is then

$$\vec{E}_T = \vec{E}_+ + \vec{E}_- \qquad \text{(P35-1)}$$

Use Coulomb's law to write down each field individually and then add them.

Write down $\vec{E}_+$ using Coulomb's law.

$$\vec{E}_+ = \left(\frac{kQ}{(x+a)^2} \right) \hat{x} \qquad \text{(P35-2)}$$

Similarly, write an expression for $\vec{E}_-$.

$$\vec{E}_- = \left(\frac{k(-Q)}{(x-a)^2} \right) \hat{x} \qquad \text{(P35-3)}$$

Add the two fields and factor out the elements common to each term.

$$\vec{E}_T = \left(\left(\frac{kQ}{(x+a)^2} \right) \hat{x} \right) + \left(\left(-\frac{kQ}{(x-a)^2} \right) \hat{x} \right)$$

(P35-4)

$$= \left((kQ) \left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2} \right) \right) \hat{x}$$

This is a perfectly correct solution. However, I asked you to consider what would happen is x became large compared to a. At first glance you might say that a could be neglected compared to x and that the field strength should be zero. This is too crude an estimate. We can help ourselves out by putting the two fractions over a common denominator. Recall that $(x+a)(x-a)=x^2+a^2$.

$$\frac{1}{(x+a)^2}-\frac{1}{(x-a)^2}=\frac{(x-a)^2-(x+a)^2}{(x+a)^2\cdot(x-a)^2}$$

$$=\frac{(x^2-2xa+a^2)-(x^2+2xa+a^2)}{((x+a)(x-a))^2} \qquad \text{(P35-5)}$$

$$=\frac{-4xa}{(x^2-a^2)^2}$$

As x becomes large compared to a the denominator becomes nearly equal to x^4. Not overlooking the x in the numerator we then have the approximate expression

$$\vec{E}_T \simeq \left(\frac{-4akQ}{x^3}\right)\hat{x} \qquad \text{(P35-6)}$$

This looks a great deal like Coulomb's law but instead of $(Q)\hat{x}$ in the numerator, we have twice what is known as the dipole moment, $(-4aQ)\hat{x}$, where the direction of this vector points from the negative to the positive charge. Much more important is the fact that the size of the field decreases as the cube of the distance instead of the square of the distance. Dipole fields are short range and weak compared to fields from unbalanced charges (monopoles). However, as there is far more balanced charge running about in our lives than unbalanced charge, these dipole fields turn out to be very important in a great many circumstances.

Think about Q20 again. We saw why the unbalanced charge was attracted to the polarized material by using Coulomb's Law. Now you can see that there will be a dipole field present at the location of the unbalanced charge and that this field, multiplied by the unbalanced charge, gives the force of attraction.

Indeed, the polarization of the neutral material can occur due to random thermal motion present in the material. This polarization lasts only a very short time and is not in some particular direction but it still results in a dipole field. If some other neutral object is nearby then it will, in turn be polarized by this transient electric field. Thus, even two electrically neutral objects will be electrically attracted to each other!

This attraction is weak and goes by the name of the Van der Waal's force. You may run into this in a chemistry course sometime in conjunction with noble gasses. The size of the force gets a $1/r^3$ dependence from each of the two polarized objects so that the force falls off as $1/r^6$. If you ever see the Lennard-Jones 6-12 potential in a chemistry class, the 6 refers to this $1/r^6$ attractive force.

P67 Having read the problem statement, there is one word in it that should leap out and smack you on the head with a large foam bat. What is it and what is its significance?[***]

Now that our heads have been smacked and we know that the sum of the forces on the small mass must be zero, we can see that this is nothing more than one of those statics problems from chapter 9. We know exactly how to work those –

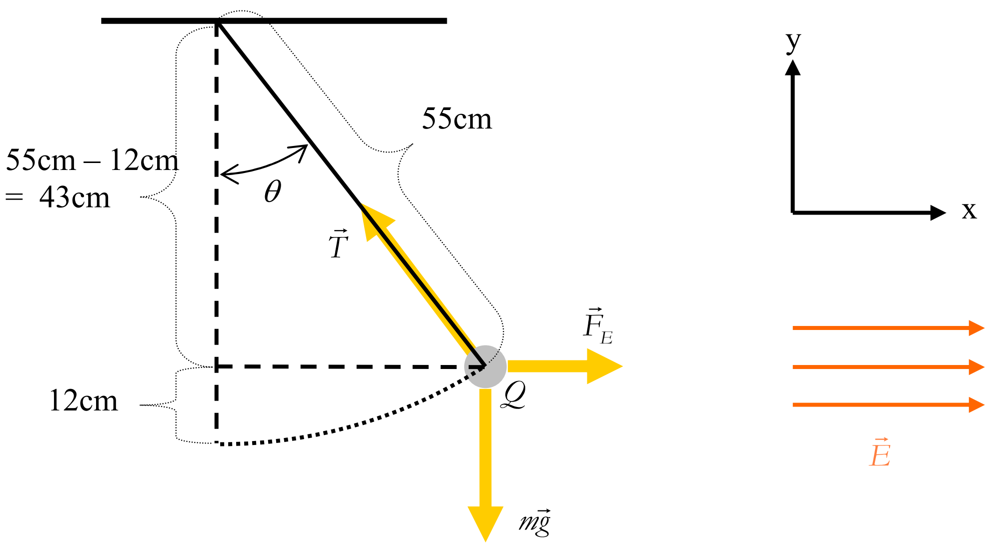
- Draw a free body diagram(s) of the object(s) showing all of the forces acting on it.
- Include a coordinate system and resolve the force vectors into components.
- Write down Newton's 2^{nd} law for each direction with the acceleration taken to be zero.
- If necessary, write down other geometrical relationships.[†††]
- If necessary, write down other physical laws.[‡‡‡]
- Solve the resulting system of equations for the requested quantity.

Let's get to work.

[***] The word is equilibrium and it significance is that the acceleration is zero. Since forces cause accelerations this, in turn, implies that the net force on the small mass is zero.

[†††] Typically these involve spatial relationships between different free body diagrams in the same problem.

[‡‡‡] You might have a spring in the problem and need Hooke's law. Here we will need F=qE.



The weight and the electric force are already aligned with the coordinate system I chose; only the tension in the string needs to be resolved into its components. From the drawing, the cosine of θ is 43/55 so θ = 38.6º.

T_x is –Tsinθ and T_y is Tcosθ.

Be careful to actually look at your diagram as you resolve your vectors into components. This rather straightforward operation creates major problems for many students simply because they are in a hurry and do not take the time to think before they write.

$$\sum F_x = 0$$

$$F_E - T\sin(\theta) = 0 \qquad \text{(P67-1)}$$

$$\sum F_y = 0$$

$$T\cos(\theta) - mg = 0 \qquad \text{(P67-2)}$$

Now I add in my additional physical law.

$$F_E = QE \qquad \text{(P67-3)}$$

Notice that the force $\vec{F}_E$ and the electric field point in the same direction in the diagram. This implies that (P67-3) should be written as it is with a positive charge being pushed in the positive x direction and a negative charge being pushed in the negative x direction.

We now have a system of three equations in three variables and we expect to be able to solve it. I will not include the details here. You should make yourself work through them. If you cannot get through this, get some help so that you develop a scheme for solving such systems of equations.

Eliminating T and F_E from (P67-1), (P67-2), and (P67-3) yields

$$Q = \frac{(mg)\tan(\theta)}{E}$$

(P67-4)

$$= 6.52 \cdot 10^{-7}\,\mathrm{C}$$